

# CRM Series in Mathematical Physics

## *Editorial Board:*

Joel S. Fieldman  
Department of Mathematics  
University of British Columbia  
Vancouver, British Columbia V6T 1Z2  
Canada  
[feldman@math.ubc.ca](mailto:feldman@math.ubc.ca)

Duong H. Phong  
Department of Mathematics  
Columbia University  
New York, NY 10027-0029  
USA  
[phong@math.columbia.edu](mailto:phong@math.columbia.edu)

Yvan Saint-Aubin  
Département de Mathématiques  
et de Statistique  
Université de Montréal  
C.P. 6128, Succursale Centre-ville  
Montréal, Québec H3C 3J7  
Canada  
[saint@math.ias.edu](mailto:saint@math.ias.edu)

Luc Vinet  
Centre de Recherches Mathématiques  
Université de Montréal  
C.P. 6128, Succursale Centre-ville  
Montréal, Québec H3C 3J7  
Canada  
[vinet@crm.umontreal.ca](mailto:vinet@crm.umontreal.ca)

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Andre D. Bandrauk • Emmanuel Lorin  
Jerome V. Moloney  
Editors

# Laser Filamentation

Mathematical Methods and Models

 Springer

*Editors*

Andre D. Bandrauk  
Faculté des Sciences  
Université de Sherbrooke  
Sherbrooke, QC, Canada

Emmanuel Lorin  
School of Mathematics and Statistics  
Carleton University  
Ottawa, ON, Canada

Jerome V. Moloney  
Arizona Center for Mathematical Sciences  
University of Arizona  
Tucson, AZ, USA

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# Preface

Optics and Photonics are now recognized as key enabling drivers that impact technologies in a multitude of areas such as communications, imaging, sensing, energy, and even health sciences. This recent declaration by NSF (USA) is based on the observation that optical and in particular laser technology has been developing at a steady rate. The invention of the laser in the 1960s for generating coherent sources of radiation led to the development of intense ultrafast pulses with time durations spanning femtoseconds ( $1 \text{ fs} = 10^{-15} \text{ s}$ ), the time scale of atomic (nuclear) motion, to the attosecond ( $1 \text{ asec} = 10^{-18} \text{ s}$ ), the time scale of electron motion in atoms and molecules. Intensities are also approaching or even exceeding the atomic unit (a. u.),  $I_0 = 3.5 \times 10^{16} \text{ W/cm}^2$ , corresponding to electric field strength  $E_0 = e/a_0^2 = 5 \times 10^9 \text{ V/cm}$  at the atomic unit of distance in the ground state of the H atom,  $a_0 = 0.0529 \text{ nm}$ . Such high fields  $E$  or equivalently intensities  $I = cE^2/8\pi$  produce ionized electrons with ponderomotive energies  $U_p(\text{eV}) = (eE)^2/(4m\omega^2) = (9.33 \times 10^{-14})I(\text{W/cm}^2)\lambda^2(\mu\text{m})$ , where  $e$ ,  $m$  are electronic charge, mass and  $\omega$ ,  $\lambda$  are the laser frequency and wavelength. Thus, lasers at current available intensities  $I = 10^{14} \text{ W/cm}^2$  and  $\lambda = 800 \text{ nm}$  generate free electrons with kinetic energies in the region of  $\sim 10 \text{ eV}$  and more. The ionization process is considered as perturbative multiphoton ionization, which for low ionization potentials  $I_p$  is transformed into a nonperturbative quantum tunnelling ionization and above barrier ionization at high intensities. For the H atom with  $I_p = 13.6 \text{ eV}$  (0.5 a. u.), tunnelling ionization is defined by the Keldysh parameter  $\gamma = (I_p/2U_p)^{1/2} < 1$  and above barrier ionization occurs for intensities exceeding the critical  $I_c = 1.4 \times 10^{14} \text{ W/cm}^2$ . Thus, at intensities  $I \sim 10^{14} \text{ W/cm}^2$  one is dealing with nonlinear response of single atoms and molecules to laser fields. In macroscopic gaseous molecular media such as the atmosphere, propagation of laser pulses leads to “Laser Filamentation” due to laser-induced nonlinear refractive indices, high-order polarizations, self-focusing, and plasma generation.

This book is based on a workshop, the first of its kind ever held in Canada, and is focused on the nonlinear theoretical and mathematical problems associated with ultrafast intense laser pulse propagation in gases and in particular in air. The chapters in this book are based on lectures by invited speakers, mainly

theorists and mathematicians and supplemented by active experimentalists who are acknowledged experts in this new field of nonlinear laser molecule interaction and propagation.

The co-organizers of the workshop especially thank the CRM (Centre de recherches mathématiques) for financial support and its staff for the administrative organization, NSF (USA) with a special grant and AFOSR for partial travel support, and the scientific publisher Springer for publishing the lectures of the invited speakers in this “International Year of Light.” We also thank Christiane Rousseau of CRM for introducing this workshop as a new activity in the international UNESCO sanctioned program MPE (Mathematics of Planet Earth) and a new direction in research on weather and climate control via current advanced laser technologies.

Sherbrooke, QC, Canada  
Ottawa, ON, Canada  
Tucson, AZ, USA  
June 2015

André D. Bandrauk  
Emmanuel Lorin  
Jerome V. Moloney

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# Contributors

**D.R. Austin** ICFO—Institut de Ciències Fotòniques, Mediterranean Technology Park, Castelldefels, Barcelona, Spain

**A.D. Bandrauk** Laboratoire de chimie théorique, Faculté des Sciences, Université de Sherbrooke, Sherbrooke, QC, Canada

Centre de recherches mathématiques, Université de Montréal, Montréal, QC, Canada

**M. Baudalet** The Townes Laser Institute, CREOL-The College of Optics and Photonics, University of Central Florida, Orlando, FL, USA

**M. Baudisch** ICFO—Institut de Ciències Fotòniques, Mediterranean Technology Park, Castelldefels, Barcelona, Spain

**J. Biegert** ICFO—Institut de Ciències Fotòniques, Mediterranean Technology Park, Castelldefels, Barcelona, Spain

ICREA—Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain

**Yuri Cher** Department of Mathematics, University of Toronto, Toronto, ON, Canada

**See Leang Chin** Centre d'optique, photonique et laser and Département de physique, de génie physique et d'optique, Université Laval, Québec, QC, Canada

**A. Couairon** Centre de Physique Théorique, CNRS, École Polytechnique, Palaiseau, France,

**Magdalena Czubak** Department of Mathematical Sciences, Binghamton University, Binghamton, NY, USA

**J. Darginavičius** Department of Quantum Electronics, Vilnius University, Vilnius, Lithuania

**A. Dubietis** Department of Quantum Electronics, Vilnius University, Vilnius, Lithuania

**Éric Dumas** Institut Fourier, Université Joseph Fourier, rue des Mathématiques, Saint Martin d'Hères, France

**M. Durand** Laboratoire d'Optique Appliquée, ENSTA ParisTech, Ecole Polytechnique, CNRS, Palaiseau, France

Townes Laser Institute. CREOL-The College of Optics and Photonics, University of Central Florida, Orlando, FL, USA

**A. Durécu** Onera-The French Aerospace Lab, BP 80100, 91123 Palaiseau Cedex, France

**D. Faccio** School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, UK

**N. Forget** FASTLITE, Centre universitaire d'Orsay, Plateau du Moulon, Orsay, France

**N. Garejev** Department of Quantum Electronics, Vilnius University, Vilnius, Lithuania

**S. Grabielle** FASTLITE, Centre universitaire d'Orsay, Plateau du Moulon, Orsay, France

**I. Gražulevičiūtė** Department of Quantum Electronics, Vilnius University, Vilnius, Lithuania

**A. Houard** Laboratoire d'Optique Appliquée, ENSTA ParisTech, Ecole Polytechnique, CNRS, Palaiseau, France

**M. Hemmer** ICFO—The Institut de Ciéncies Fotóniques, Mediterranean Technology Park, 08860 Castelldefels, Barcelona, Spain

**A. Jarnac** Laboratoire d'Optique Appliquée, ENSTA ParisTech, Ecole Polytechnique, CNRS, Palaiseau, France

**Jingjing Ju** State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai, China

Centre d'optique, photonique et laser and Département de physique, de génie physique et d'optique, Université Laval, Québec, QC, Canada

**V. Jukna** Centre de Physique Théorique, École Polytechnique, Palaiseau Cedex, France

**Miroslav Kolesik** College of Optical Sciences, University of Arizona, Tucson, AZ, USA

Arizona Center for Mathematical Sciences, University of Arizona, Tucson, AZ, USA

**David Lannes** Institut de Mathématiques de Bordeaux, Université de Bordeaux & CNRS UMR, cours de la Libération, France

**Ruxin Li** State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai, China

**K. Lim** Townes Laser Institute, CREOL-The College of Optics and Photonics, University of Central Florida, Orlando, FL, USA

**Peng Liu** State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai, China

**Y. Liu** Laboratoire d'Optique Appliquée, ENSTA ParisTech, Ecole Polytechnique, CNRS, Palaiseau, France

**E. Lorin** School of Mathematics and Statistics, Carleton University, Ottawa, ON, Canada

Centre de recherches mathématiques, Université de Montréal, Montréal, QC, Canada

**M. Lytova** School of Mathematics and Statistics, Carleton University, Ottawa, ON, Canada

**D. Majus** Department of Quantum Electronics, Vilnius University, Vilnius, Lithuania

**E. McKee** Townes Laser Institute. CREOL-The College of Optics and Photonics, University of Central Florida, Orlando, FL, USA

**Jerome V. Moloney** College of Optical Sciences, University of Arizona, Tucson, AZ, USA

Arizona Center for Mathematical Sciences, University of Arizona, Tucson, AZ, USA

Department of Mathematics, University of Arizona, Tucson, AZ, USA

**A. Mysyrowicz** Laboratoire d'Optique Appliquée, ENSTA ParisTech, Ecole de Polytechnique, CNRS, Palaiseau, France

**Alan C. Newell** Department of Mathematics, University of Arizona, Tucson, AZ, USA

**Paris Panagiotopoulos** College of Optical Sciences, University of Arizona, Tucson, AZ, USA

Arizona Center for Mathematical Sciences, University of Arizona, Tucson, AZ, USA

**M. Richardson** The Townes Laser Institute, CREOL-The College of Optics and Photonics: CREOL and FPCE, University of Central Florida, Orlando, FL, USA

**F. Silva** ICFO—Institut de Ciències Fotòniques, Mediterranean Technology Park, 08860 Castelldefels, Barcelona, Spain

**Catherine Sulem** Department of Mathematics, University of Toronto, Toronto, ON, Canada

**J r mie Szeftel** Laboratoire Jacques-Louis Lions, Universit  Pierre et Marie Curie, Paris, France

**G. TamoŐauskas** Department of Quantum Electronics, Vilnius University, Vilnius, Lithuania

**A. Thai** ICFO—Institut de Ci ncies Fot niques, Mediterranean Technology Park, 08860 Castelldefels, Barcelona, Spain

**G. Valiulis** Department of Quantum Electronics, Vilnius University, Vilnius, Lithuania

**Tie-Jun Wang** State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai, China

Centre d'optique, photonique et laser and D partement de physique, de g nie physique et d'optique, Universit  Laval, Qu bec, QC, Canada

**Patrick Townsend Whalen** College of Optical Sciences, University of Arizona, Tucson, AZ, USA

Department of Mathematics, University of Arizona, Tucson, AZ, USA

**Zhizhan Xu** State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai, China

**Shuai Yuan** Centre d'optique, photonique et laser and D partement de physique, de g nie physique et d'optique, Universit  Laval, Qu bec, QC, Canada

Shanghai Key Laboratory of Modern Optical System, Engineering Research Center of Optical Instrument and System, University of Shanghai for Science and Technology, Shanghai, China

State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai, China

**Heping Zeng** Shanghai Key Laboratory of Modern Optical System, Engineering Research Center of Optical Instrument and System, University of Shanghai for Science and Technology, Shanghai, China

State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai, China