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Essential Partial Differential Equations

Analytical and Computational Aspects

Springer

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In mathematics you don't understand things. You just get used to them.

John von Neumann

"Begin at the beginning", the King said, very gravely, "and go on till you come to the end: then stop".

Lewis Carroll, *Alice in Wonderland*

Preface

This textbook is a self-contained introduction to the mathematical aspects of partial differential equations. The book is aimed at undergraduate students studying for a mathematics degree. Selected chapters could also be of interest to engineers and scientists looking to develop an understanding of mathematical modelling and numerical analysis.

The material is organised into 13 chapters, with roughly equal emphasis placed on *analytical* and *numerical* solution techniques. The first four chapters provide a foundation for the study of partial differential equations. These chapters cover physical derivation, classification, and well-posedness. Classical solution techniques are discussed in Chaps. 8 and 9. Computational approximation aspects are developed in Chaps. 6 and 10–12. A clear indication is given in each of these chapters of where the basic material (suitable perhaps for a first course) ends and where we begin to probe more challenging areas that are of both a practical and theoretical interest. The final chapter defines a suite of projects, involving both theory and computation, that are intended to extend and test understanding of the material in earlier chapters.

Other than the final chapter, the book does not include programming exercises. We believe that this strategy is in keeping with the aims and objectives of the SUMS series. The availability of software environments like MATLAB (www.mathworks.com), Maple (www.maplesoft.com) and Mathematica (www.wolfram.com) means that there is little incentive for students to write low-level computer code. Nevertheless, we would encourage readers who are ambitious to try to reproduce the computational results in the book using whatever computational tools that they have available.

Most chapters conclude with an extensive set of exercises (almost 300 in all). These vary in difficulty so, as a guide, the more straightforward are indicated by$^{☆}$ while those at the more challenging end of the spectrum are indicated by$^{★}$. Full solutions to all the exercises as well as the MATLAB functions that were used to generate the figures will be available to authorised instructors through the book's website (www.springer.com). Others will be able to gain access to the solutions to odd-numbered exercises though the same web site.

Distinctive features of the book include the following.

1. The level of rigour is carefully limited—it is appropriate for second-year mathematics undergraduates studying in the UK (perhaps third or fourth year in the USA). The ordering of topics is logical and new concepts are illustrated by worked examples throughout.
2. Analytical and numerical methods of solution are closely linked, setting both on an equal footing. We (the authors) take a contemporary view of scientific computing and believe in mixing rigorous mathematical analysis with informal computational examples.
3. The text is written in a lively and coherent style. Almost all of the content of the book is motivated by numerical experimentation. Working in the "computational laboratory" is what ultimately drives our research and makes our scientific lives such fun.
4. The book opens the door to a wide range of further areas of study in both applied mathematics and numerical analysis.

The material contained in the first nine chapters relies only on first-year calculus and could be taught as a conventional "introduction to partial differential equations" module in the second year of study. Advanced undergraduate level courses in mathematics, computing or engineering departments could be based on any combination of the early chapters. The material in the final four chapters is more specialised and, would almost certainly be taught separately as an advanced option (fourth-year or MSc) entitled "numerical methods for partial differential equations". Our personal view is that numerical approximation aspects are central to the understanding of properties of partial differential equations, and our hope is that the entire contents of the book might be taught in an integrated fashion. This would most probably be a double-semester (44 hours) second- (or third-) year module in a UK university. Having completed such an integrated (core) course, students would be perfectly prepared for a specialist applied mathematics option, say in continuum mechanics or electromagnetism, or for advanced numerical analysis options, say on finite element approximation techniques.

We should like to extend our thanks to Catherine Powell, Alison Durham, Des Higham and our colleagues at Manchester and Dundee, not to mention the many students who have trialled the material over many years, for their careful reading and frank opinions of earlier drafts of the book. It is also a pleasure to thank Joerg Sixt and his team at Springer UK.

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