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Editors

Quantization, PDEs, and Geometry

The Interplay of Analysis and Mathematical Physics

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Geodesics in Geometry with Constraints and Applications

I. Markina

Introduction

During the long history of natural sciences it has turned out that various concepts originating in physics became a rich source for mathematical research. In particular this is the case in the area of analysis and an embedding of such physical ideas into a more abstract mathematical framework can be expected to provide a deeper structural insight. Conversely this mathematical treatment may lead to new applications and achievements in physics. The present volume aims to collect four different topics in modern mathematics that are under on-going research and closely linked to applications in physics. Written by leading experts of the field these papers have an introductory character and are accessible to non-specialists. The authors attached importance to a detailed introduction and clear motivation of the subject. Most of the proofs are given or, when too long, cited from the literature. Finally, open questions and some of the most recent approaches to the different topics are discussed.

The article by Todor Gramchev is concerned with linear and semilinear pseudodifferential equations on Euclidean space $\mathbb{R}^n$. Besides the usual local theory, here global aspects become important. It is one basic observation that solutions of such equations often decay exponentially at infinity. To obtain sharp results in this direction, solutions of the equations under study are sought in Gelfand–Shilov spaces. Semilinear equations are then regarded as perturbations of the corresponding linear ones, and the linear equations are investigated with the help of various pseudodifferential calculi on $\mathbb{R}^n$, with global estimates on the amplitude functions and special control of the implied constants in these estimates. A number of typical applications is also mentioned like exponential location of eigenfunctions of stationary Schrödinger operators with polynomially growing potentials in phase space and the construction of traveling wave solutions of certain nonlinear equations.

The paper by Miroslav Engliš aims to give a flavour of two quantization theories in physics: the Berezin and the Berezin–Toeplitz quantization. As an illuminating example the cases where the quantized domains are the entire complex space $\mathbb{C}^n$ and the unit disc are studied by employing Toeplitz operators on the Fock and the Bergman space, respectively. Generalizing this concept the quantization of a domain $\Omega \subset \mathbb{C}^n$ equipped with a Kähler form ω and corresponding Poisson bracket is explained. Finally, the author sketches the proof of the existence theorem for Berezin and Berezin–Toeplitz quantization of smoothly bounded strictly pseudoconvex domains which is based on the Boutet de Movel theory and Fefferman's expansion of the Szegő kernel.

The article by Andrew Comech discusses the soliton resolution conjecture by the example of a nonlinear wave equation in one space dimension. This is in fact one of the main conjectures in the field of dispersive equations, and it is wide open in space dimensions two and higher. Here, the general motivation behind addressing such questions is explained. Then the Klein–Gordon equation in one space dimension with a nonlinearity located at one point is studied. This equation is globally well posed in the energy space, and the soliton resolution conjecture asserts in this case that any finite energy solutions converges as $t \to \infty$ towards a global attractor which is entirely composed of solitary waves. A detailed proof of this result is provided.

The contribution by Irina Markina provides a glimpse into the area of subriemannian geometry and its various applications. In particular, there are close connections to classical mechanics, CR manifolds or geometric control theory. Starting from a subriemannian metric a Hamiltonian formalism is explained that produces geodesics under non-holonomic constraints. The author mentions a variety of concrete examples in which the subriemannian geometry originates from a Lie group structure or is induced via a principle fibre bundle. Subsequently the kinematic system of a manifold rolling on another manifold without twisting and slipping is addressed in the framework of subriemannian geometry. The last part of the paper discusses subriemannian structures on infinite-dimensional Lie groups and the problem of controllability.

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