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Nicolás Rubido

Energy Transmission and Synchronization in Complex Networks

Mathematical Principles

Doctoral Thesis accepted by
the University of Aberdeen, UK

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To my family,
including "el Gonxa"

Supervisor's Foreword

The scientific motivations for this Ph.D. work were to understand in analytical fashion the complex relationship between network structure (topology and interacting function) and its behavioural manifestations (transmission of energy and synchronisation) in two closely related complex systems: power systems and networks of interacting phase oscillators. The technological motivation behind the Ph.D. work was to contribute mathematically towards the creation of a power system that is capable of delivering power as demanded, in a self-controlled, stable (i.e. resilient to external perturbations, structural modifications, and cascade effects, even without any active control) and smarter way (i.e, allowing the decentralisation of large power stations into small fluctuating renewable energy-sources). This book is therefore aimed at any researcher or post-graduate student with a good mathematical background who is either interested in making contributions to the field of synchronisation in complex networks or in the field of stability and optimisation of power systems.

Dr. Nicolás Rubido proposed brilliant analytical results to understand three tractable problems. In Sect. 3.1, he shows how the distribution of voltages and electric currents are related to the network's structure in simple models of power systems: DC/AC flow networks. In Chap. 4, he shows how synchronisation depends on the network's characteristics in a paradigmatic model of complex systems, namely, networks of interacting phase oscillators. Finally, in Sect. 3.2, he goes on to analytically demonstrate how the transmission of power is related to both synchronisation and the network's structure, in a more realistic model of a power system—the Swing equations—which treats a power system as a network of power generators.

All the results presented in this book can be extended to treat other complex systems with similar descriptions to those treated here. In particular, Dr. Rubido's results in DC/AC flow networks can be used to analytically calculate loads (potentials) and physical flows (a physical quantity passing a cross section per time unit) in any conservative network, i.e. where the input flow equals the output flow and whose loads' difference and physical flows along an edge are linearly related.

His results are therefore applicable to understand cracks in materials, river flow networks, traffic flow networks and other systems. On the other hand, Dr. Rubido's results in networks of phase oscillators and in the Swing equations can be extended to other complex networks, as long as the problem to be tackled is related to determining conditions for the stability of the frequency synchronous state and its phase distribution. Frequency stability in power systems is important to maintain reliability of the machines (generators and consumers) and optimal distribution (minimal dissipation). Broad phase distributions are necessary to transmit power. Thus, his results can be used to better understand synchronisation of rhythms in natural and technological systems.

To assist the reader who is non-familiar with the theory of graphs, the spectral theory, or the theory of complex networks, and to fully understand Dr. Nicolás Rubido's elegant mathematical derivations and their insights, and to learn how to extend his results to other complex systems he gives, in Chap. 2, a comprehensive review on these topics. Anyone wishing to treat natural complex systems in a stricter mathematical fashion is recommended to read this chapter.

I would like to conclude this foreword by directing the reader on a tour of some of the highlights of Rubido's results. These include: their comprehensive analysis towards fundamental and technological implications, such as the equation to calculate analytically the electric flow in AC/DC network as a function of the eigenvalues and eigenvectors of the network structure [Eq. (3.10)]; the demonstration that maximal electric flows are not altered even if an arbitrary number of intermittent input sources exists [Eq. (3.20)] the pathway to understand how someone who has little information about the network structure can grow a DC/AC network without affecting the maximal flows [using Eqs. (3.21)–(3.24)] the three conditions [Eqs. (3.43)–(3.45)] that guarantee frequency stability in the Swing equation, and, finally, the demonstration that to guarantee the existence of solutions [Eq. (4.16) and Table 4.1] and their stability [Eq. (4.23) and Table 4.2] in a network of phase oscillators, not only the symmetry of the network structure is important, but the first derivative values and parity of the coupling function also need to be taken into account. In particular, the latter result, as pointed out by Dr. Rubido in this book, opens a new avenue of research based on the relevance of coupling functions for the behaviour of complex systems.

Aberdeen, UK
August 2015

Dr. Murilo S. Baptista

Abstract

Understanding how the transmission of energy between the providers (such as nuclear power stations, renewable resources, or any type of supplying entity) and the consumers (such as factories, homes, or any type of demanding entity) depends on the structure of the inter-connections between them and on their dynamical behaviour is of paramount importance for the design of power-grid systems that are resilient to failures, e.g. failures due to structural modifications or energy fluctuations. In this thesis, we derive the implicit relationship between the structure and the behaviour that flow and power networks have, namely, the mathematical principles behind the transmission of energy in complex networks. From our novel derivations, we determine exact and approximate strategies to create self-controlled and stable systems (i.e. resilient to failures without the need for external controllers) that have an optimal (i.e. with less cost and power dissipation) and smart (i.e. allowing the decentralisation of large power stations to smaller fluctuating renewable resources) energy distribution. Moreover, not only we achieve analytical solutions for problems that usually require extended numerical analysis, but we also propose a change in the analysis viewpoint of complex systems, namely, systems composed of many dynamically interacting units forming a network. We show that in order to explain the emergent behaviour in these systems, instead of focusing on the network structure of the interactions, we should focus on the functional form of the interactions. In particular, we derive a general framework to study the existence and stability of emergent collective behaviour in networks of interacting phase oscillators, namely, the mathematical principles behind the synchronisation in complex networks. The numerous breakthrough results in this thesis are expected to be of aid for engineers to design smarter and more resilient power-grid systems, as well as to scientists dealing with emergence phenomena in complex systems.

Acknowledgments

This thesis is the result of the cumulative effort, not only from these past three years, but that of a lifetime. Numerous people played a major role in my development as a Physicist, from my childhood to my adulthood. Consequently, I see no better way of starting this thesis than by acknowledging how important they have been to me and how thankful I am to them for being part of my life.

I start by thanking my Ph.D. supervisors, Celso Grebogi and Murilo S. Baptista. Without them I would have been unable to accomplish this thesis. They made me feel honoured to be their student. Not only they were a huge source of inspiration, learning and admiration during the Ph.D., but both have become dear friends and colleagues. Especially Murilo, who cheerfully encouraged every achievement I made or research plan I undertook and always opened his office door willing to pursue endless debates (and with endless patience!). I learnt from you both enormously. My dear friends, my debt to you is eternal.

The undertaking of a Ph.D. thesis takes away a big part of the daily life. However, there is still room for much outside of it. I cherish this part of my life greatly. During this part, I got to know Scotland's traditions and wonders. These were all thanks to my friends from the ICSMB and Aberdeen city. I thank you all! Besides the people in Aberdeen, I also received help from the friends I made in Terrassa, at the DONLL group of the Universidad Politécnica de Catalunya (UPC) in Spain thanks to Cristina Masoller, who has been a great friend, colleague and mentor. In Terrassa, I met some incredible people along the years that contributed in different ways to my thesis and who I sincerely thank as well.

The accomplishment of this thesis was possible because I had the chance to come to the University of Aberdeen to pursue a Ph.D. thanks to the Scottish Universities Physics Alliance (SUPA) scholarship prize. However, I was only able to obtain the prize because of the background knowledge I gained. This knowledge I acquired from my undergraduate and post-graduate education and work experience at the Instituto de Física, Facultad de Ciencias (IFFC), Universidad de la República (UdelaR), Montevideo, Uruguay, from 2002 until 2011. Besides the eternal appreciation and gratitude to the IFFC and all the people that I met through those years, I especially thank the dear friends I made while struggling with the undergraduate and post-graduate courses and exams. In particular, to Javier Brum, Lucía Duarte, Luis Pedro García Pintos, Daniel Freire, Sofía Favre, Marcela Peláez, Sebastián Bruzzone, Nicasio Barrere, Andrés Melo, Cesar Voulgaris, Pablo Pais and Bárbara Fraygola. Thanks to you all, I discovered a passion for research. During the last two years at the IFFC, I pursued my M.Sc. degree under the supervision of Cecilia Cabeza and Arturo C. Martí. They both were incredible mentors to me and dear friends. I am glad and honoured to be still an active part of their research group.

Before my academic life at the UdelaR started, I lived in Salto, my home town. I am extremely thankful to my friends from that time; the ones that grew up together with me. In particular, I thank my dearest and closest friends, Gonzalo Frakksoni, Federico Castrillón, Juan Pedro Monetta, Daniel Sebastián Mazzoncini, Juan Martín Rinaldi and Gabriel Silva. If I have achieved so much so far, it has only been because you were always there for me. Your friendship means the world to me.

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> […] para vos no es novedad
> que el mundo
> y yo
> te queremos de veras
> pero yo siempre un poquito más que el mundo.

Extract from the poem by Mario Benedetti, "Como Siempre", *El amor, las mujeres y la vida*, p. 3 (1993).

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Notations

Observations on Notations

We observe that the notations we use in this thesis are as follows. We denote a *set* by italic capital letters, such as $\mathcal{G}$ for the graph set or $\mathcal{E}$ for the set of edges in a graph. Contrary, a *matrix* is indicated by bold capital letters, such as the Laplacian matrix $\mathbf{G}$, and the *matrix elements* by square brackets, namely, $[\mathbf{G}]_{ij} \equiv G_{ij}$ is the *ij*th entry. The *Kronecker-delta function*, δ_{ij}, is widely used in this thesis and is the function that fulfils $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ otherwise ($i \neq j$). In other words, the Kronecker-delta function is simply the identity matrix, $\mathbf{I}$, elements. In particular, if the Laplacian matrix is derived from a generic matrix $\mathbf{A}$, then, the derived Laplacian matrix ($\mathbf{G} = \mathbf{D} - \mathbf{A}$, where $[\mathbf{D}]_{ij} \equiv \delta_{ij} \sum_{k=1}^{N} A_{ik}$) is noted as $\mathbf{G}(\mathbf{A})$ or $\mathbf{G}(A)$. A column *vector* is noted by $\vec{x}$, hence, its *i*th element is given by $[\vec{x}]_i \equiv x_i$. For example, the *eigenvalues* and *eigenvectors* of a Laplacian matrix we note as λ_n and $\vec{\psi}_n$. We use i, j, k, l, m and n as index for summations.

Notations for Methods

$\mathcal{G}$	A graph set
$\mathcal{V}$	Set of nodes in a graph
$\mathcal{E}$	Set of edges in a graph
$\mathcal{W}$	Set of weights for the edges in a graph
$\#$	Cardinality of a set, i.e. number of elements
$\emptyset$	Empty set
$\cup$	Union of sets
$\cap$	Intersection of sets
$\equiv$	Identity definition, i.e. if $x \equiv y$, we define x to be identical to y
$\mathbb{N}$	Natural numbers
$\mathbb{R}$	Real numbers

$\mathbb{C}$	Complex numbers
$\mathbb{M}_{N\times N}$	Matrices of size $N \times N$
δ_{ij}	Kronecker-delta function
$\mathbf{G}$	Laplacian matrix
$\mathbf{X}$	Pseudo-inverse Laplacian, i.e. Moore-Penrose matrix
$\mathbf{A}$	Adjacency matrix
$\mathbf{W}$	Weighed adjacency matrix
$\mathbf{D}$	Diagonal matrix of node degrees
$\mathbf{D}_w$	Diagonal matrix of weighed node degrees
d	Degree of a node
dw	Weighed degree of a node
L	Path length
Υ	Equivalent resistance matrix
Kf	Kirchhoff index of a graph
λ	An eigenvalue of a Laplacian matrix
$\vec{\psi}$	A column eigenvector of a Laplacian matrix
Λ	Diagonal matrix of eigenvalues
$\mathbf{P}$	Matrix of column eigenvectors
*	Conjugate operation
^{T}	Transpose operation
$\dagger$	Adjoint operation, i.e. transpose plus conjugate operations (e.g. $\mathbf{P}^\dagger \equiv \mathbf{P}^{T^*}$)
$\mathbf{P}^{-1} = \mathbf{P}^\dagger$	Unitary matrix, i.e. a matrix whose inverse is its adjoint
$\mathbf{P}^{-1} = \mathbf{P}^T$	Orthogonal matrix, i.e. a matrix whose inverse is its transpose
$\tilde{z}$	z is complex valued
$\mathbf{I}$	Identity matrix ($I_{ij} = \delta_{ij}$)
$\mathbf{J}$	Unitary matrix ($J_{ij} = 1$)

Notation for Transmission of Energy

$\vec{A}$	Real part of the net flow vector
$\vec{B}$	Imaginary part of the net flow vector
ρ	Conductance matrix
σ	Susceptance matrix
ΔV_{kl}	Voltage difference between nodes k and l
I_{kl}	Current between nodes k and l
$\mathbf{Y}$	Admittance matrix
β	Dissipative coefficient
$\mathbf{R}$	Dissipation matrix for the transmission lines of power-grids (dependent on ρ)

P	Oscillation matrix for the transmission lines of power-grids (dependent on σ)
θ	Instantaneous rotor angle variable
$\vec{\epsilon}$	Perturbation vector, i.e. instantaneous deviations from any particular trajectory of the rotor angle variable
J	Jacobian matrix of the perturbation dynamics
α	A characteristic exponent of **J**, namely, an eigenvalue

Notation for Synchronisation

ϕ	Phase-angle variable in a non-rotating framework
θ	Phase-angle variable in a rotating framework
ω	Rotational angular frequency in non-rotating framework
$\delta\omega$	Rotational angular frequency in rotating framework
Ω	Angular frequency of the rotating framework
σ	Coupling strength
Γ	Coupling function
W	Coupling topology matrix, namely, the network weighed adjacency matrix