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Igor V. Shevchuk

Modelling of Convective Heat and Mass Transfer in Rotating Flows

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Igor V. Shevchuk
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Fellbach-Schmiden
Germany

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To my wife Nataliya and sons Vladimir,
Aleksandr and Nikolay

Preface

This monograph was conceived as an overview of the potential and achievements of the analytical and numerical modelling of convective heat and mass transfer in different types of rotating flows. Flow rotation can be a consequence of (i) *system rotation*, (ii) *flow swirl* imposed by so-called swirl generators and (iii) *curvature of surfaces* or larger segments of the geometry such as turns, bends, curved connections, etc. Rotation, swirl or curvature-induced volume forces often referred to as centrifugal and Coriolis forces can significantly affect the flow pattern, as well as the heat and mass transfer rate.

Rotating flows arise in numerous scientific and engineering applications. As practical examples one can mention turbomachinery, energy systems, automotive engineering, aerospace engineering, medical equipment, processing engineering and many others. One of the important scientific applications is a rotating disk electrode involved in experimental determination of the diffusion coefficient in electrolytes. A cone-plate or a cone-disk device, which includes a fixed disk and a rotating cone that touches the disk by its apex, is widely known in measurements of the viscosity coefficient of liquids.

One can mention several books, which elucidate many aspects of the physics and provide quantification for many parameters of rotating flows. These are classical books of L.A. Dorfman "Hydrodynamic Resistance and the Heat Loss of Rotating Solids" (Oliver and Boyd, Edinburgh, UK, 1963) and V.G. Levich "Physicochemical Hydrodynamics" (Prentice-Hall, Inc., Englewood Cliffs, N.J., 1962), which for many decades were desktop books for specialists in the fields of convective heat and mass transfer in rotating disk systems. The fundamental review monograph of J.M. Owen & R.H. Rogers "Flow and Heat Transfer in Rotating-Disc Systems" (Research Studies Press Ltd., UK, 1989 & 1995) summarized results of experimental investigations and theoretical modelling in the area of secondary air cooling systems of gas turbines, including rotor−stator systems and rotating cavities formed by parallel co-rotating disks. The book "Heat Transfer and Fluid Flow in Rotating Coolant Channels" (Research Studies Press, J. Wiley and Sons, 1981) by W.D. Morris is devoted mostly to experimental investigations of the hydraulic resistance and average heat transfer in channels rotating about a parallel axis. In the recent book “Rotating flow” (Elsevier Inc., Amsterdam etc., 2011), P.R.N. Childs shed light on the basic theory of rotating flows and contributed to the development of the integral methods for rotating-disk systems, including rotor-stator configurations and rotating cavities.

During the past decades, methods of experimental and theoretical investigations of scientific and practical problems of convective heat and mass transfer, which the aforementioned books deal with, have made considerable progress. This resulted in obtaining new accurate experimental and numerical results for a series of rotating geometries studied before and emerging during the past years. As a result, the analysis and generalizations of the experimental and numerical data provided in the aforementioned books are often insufficient. Integral methods have been rather successfully applied to several rotating-disk geometries. However, theoretical model assumptions underlying the known integral methods demonstrated their restricted capabilities in light of the newly obtained experimental and numerical data. A powerful modelling technique based on the exact self-similar solutions of the Navier–Stokes and energy equations appeared to be insufficiently developed for a few rotating disk geometries, where appropriate self-similar forms of the solutions have not been derived. In addition, a few important scientific and practical problems were not touched in the above-mentioned books: (a) transient conjugate heat transfer; (b) uniform orthogonal flow impingement onto a disk; (c) fluid flow, heat and mass transfer in a small gap between a rotating disk and/or a cone that touches the disk by its apex; (d) convective heat and mass transfer at Prandtl and Schmidt numbers, both moderately larger than unity with the application to the experimental technique based on naphthalene sublimation in air, and much larger than unity with the application in electrochemistry.

All said, the above became an incentive for me to undertake investigations that were summarized in the form of a book by Shevchuk I.V. “Convective Heat and Mass Transfer in Rotating Disk Systems (Springer Verlag, Berlin, Heidelberg, 2009). Since then, I have conducted new studies on the subject of rotating flows published as a series of research papers. In the same time, the international scientific community has also contributed much to this research area, which provided valuable material for validation and corroboration of the models and numerical results presented in my book.

In comparison with the previous book, my new monograph outlines the further progress in the integral methods, self-similar and analytical solutions for the problems of convective heat and mass transfer in rotating-disk systems validated through extensive comparisons with the experimental data including those that have been published during the past six years. Most part of the new monograph is devoted to system rotation-induced fluid flows and deals with several rotating-disk geometries. Swirl flows were also modelled in some of these geometries. In addition, the scope of the new monograph was extended to cover other types of rotating flows such as those in (a) the channels rotating around a parallel axis, and (b) the two-pass ribbed channels with 180° bends. These studies provide examples of design optimization of air cooling systems of the rotors of electrical motors and gas turbine blades, respectively.

The present book consists of nine chapters. The book is mainly focused on convective heat transfer in air flow, with the exception of Chap. 6, which deals with heat and mass transfer at Prandtl or Schmidt numbers larger than unity.

Chapter 1 depicts geometries studied in this book, outlines forces influencing the flow and presents a general mathematical description in the form of momentum, continuity, energy and convective diffusion equations written in a vector form, Cartesian and cylindrical polar coordinate systems.

In Chap. 2, the general mathematical description is adapted to rotating disk systems. The chapter contains also an overview of the existing methods of solution, the integral method developed by the author, and a general analytical solution for the turbulent boundary layer flow and heat transfer in rotating-disk systems obtained using this method.

Chapter 3 is devoted to steady-state and unsteady heat transfer of a single rotating disk. As demonstrated here, the present integral method is significantly more accurate and incorporates a wider variety of thermal boundary conditions than other integral methods. Chapter 3 critically overviews the most reliable experimental data for transitional flow, provides recommendations for the calculation of average heat transfer of an *entire* disk and briefly outlines some important aspects of laminar transient heat transfer.

In Chap. 4, results of the analytical and numerical modelling of external flow over a rotating disk and outward flow between parallel co-rotating disks are described and compared with experimental data. In particular, Chap. 4 presents solutions for the cases of (a) disk rotation in a fluid subject to solid-body rotation, (b) accelerating non-rotating radial flow and (c) centrifugal swirling radial flow in a gap between parallel co-rotating disks.

Chapter 5 focuses on laminar flow, heat and mass transfer between a disk and a cone that touches the disk with its apex. It comprises such geometries as "rotating cone—stationary disk", "rotating disk—stationary cone", "co-rotating or contra-rotating disk and cone" and "non-rotating conical diffuser". Novel is the section describing effects of the Prandtl and Schmidt numbers, as well as a review of the relevant recently published works.

In Chap. 6, results of different authors for the problems of convective heat and mass transfer for the Prandtl and Schmidt numbers larger than unity are critically analysed and generalized. Chapter 6 presents original theoretical models of the author developed for naphthalene sublimation in air and electrochemical problems. In the integral method of the author, effects of large Prandtl and Schmidt numbers are taken into account.

Chapter 7 describes results of the CDF modelling of convective heat transfer in pipes rotating around a parallel axis including effects of the flow angle of attack at the inlet to the pipe, as well as influence of the cross-section geometry (circular or elliptic pipes).

In Chap. 8, original results of the simulation and optimization of convective heat transfer in the varying aspect ratio two-pass internal ribbed cooling channels with 180° bends are outlined and analysed from a single viewpoint.

Chapter 9 presents overall conclusions to the material presented in the book.

The present book is an official publication of my habilitation thesis (Habilitationsschrift) prepared during my work as a university lecturer at the Institute of Aerospace Thermodynamics (ITLR), University of Stuttgart, Germany and successfully presented at the Faculty of Aerospace Engineering and Geodesy, University of Stuttgart on 24 April 2015. I would like to deeply thank the main referee of my habilitation thesis, Director of the ITLR Prof. Dr.-Ing. habil. Bernhard Weigand for his strong support of my aspiration to successfully accomplish this work, as well as for his numerous valuable advices and fruitful discussions. I would also like to warmly thank the co-referees, Professor D.Phil. Peter R.N. Childs (Imperial College London, UK), Professor Dr. Andrey V. Kuznetsov (North Carolina State University, U.S.A.) and Professor Dr.-Ing. habil. Yuri B. Zudin (National Research Center "Kurchatov Institute", Moscow, Russia) for reviewing my habilitation thesis and for participation in the habilitation process.

I have obtained results included in this work during two decades of research studies conducted in collaboration with different research institutions in Germany, USA, UK, France, Sweden and Ukraine. I would like to thank all my colleagues, with whom I collaborated during that time, for their contribution, useful advices and friendly discussions. The Research Fellowship of Alexander von Humboldt Foundation (Germany), which enabled my research stay at Technische Universität Dresden in 2003–2005 and summarizing a large part of the material used in the present book, is also gratefully acknowledged.

I am very thankful to my family for their invaluable constant support and understanding during my preparation of the habilitation thesis.

Stuttgart, Germany Igor V. Shevchuk

Contents

Nomenclature

a	Thermal diffusivity; (m^2/s)
a	Radial velocity gradient on the outer boundary of the boundary layer, Eq. (2.27); (1/s)
a	Semi-major axis of an ellipse; (m)
$A = ad_j/V_j$	Non-dimensional radial velocity gradient on the outer boundary of the boundary layer; (–)
AR = H/W	Aspect ratio; (–)
b	Outer radius of a disk; (m)
b	Semi-minor axis of an ellipse; (m)
$Bi_1 = \alpha_1 b/\lambda_w$	Biot number at a cylindrical surface of a disk; (–)
$Bi = 0.5\alpha s/\lambda_w$	Biot number at a flat surface of a disk; (–)
$Bi_2 = 0.5\alpha_2 s/\lambda_w$	Biot number at a flat surface of a disk; (–)
C	Concentration; (mol/m^3)
$c_f/2 = \tau_w/(\rho V_*)^2$	Surface friction coefficient; (–)
$C_M = 4M/(\rho\omega^2 b^5)$	Moment coefficient of two flat sides of a rotating disk; (–)
c_p	Isobaric specific heat; (J/(kg K))
$C_w = \dot{m}/(\mu b)$	Non-dimensional radial mass flowrate through a cavity between two rotating disks; (–)
$c_{0^*} = (T_{w,i} - T_\infty)_{n_*=0}$	Constant temperature difference on the surface with $T_{w,i}$ = const. and T_∞ = const.; (K)
D	Disk diameter; (m)
D	Diameter of the circular pipe; (m)
D_j	Nozzle diameter; (m)
D_m	Diffusion coefficient; (m^2/s)
$D_h = 4S/P_e$	Hydraulic diameter (arbitrary cross-section); (m)
$D_h = \dfrac{2HW}{H+W}$	Hydraulic diameter (rectangular channel); (m)
$D_e = \sqrt{4S/\pi}$	Equivalent diameter; (m)
e	Rib height; (m)

$\boldsymbol{F}$	Mass force per unit volume (boldface denotes vector parameter); (N/m^3)
F_x, F_y, F_z	Mass force components in Cartesian coordinates (per unit volume); (N/m^3)
F_r, F_φ, F_z	Mass force components in cylindrical polar coordinates (per unit volume); (N/m^3)
F, G, H, P	Self-similar functions, Eq. (2.26); (–)
F_0, G_0, H_0	Self-similar velocity components, free rotating disk (subscript "0"), Eq. (2.26) at $\beta = 0$, $N = 0$; (–)
$\mathrm{Fo} = 4a_w t/s^2$	Fourier number; (–)
$F_\mathrm{t}(t) = \dfrac{T_w(t,r) - T_\infty}{T_{w,i}(r) - T_\infty}$	Non-dimensional disk surface temperature in the unsteady heat transfer problem; (–)
$f = \Delta p D_h/(0.5\rho \bar{V}^2 L)$	Friction factor in a single channel/pipe; (–)
$f = \Delta p D_{hi}/(0.5\rho U_i^2 L)$	Friction factor in a two-pass channel; (–)
f_0	Friction factor, Blasius Eq. (7.11) or McAdams Eq. (8.1) (–)
g	Acceleration of gravity; (m/s^2)
$h = r\ \mathrm{tg}\gamma$	Height of a conical gap; (m)
h_j	Nozzle-to-disk distance; (m)
$H = b/(0.5s)$	Parameter in Eq. (4.21); (–)
H	Eccentricity; (m)
H	Height of a rectangular channel, m; (m)
I	Turbulence intensity; (–)
$\boldsymbol{j}$	Acceleration of a mass force; (m/s^2)
k	Turbulent kinetic energy per unit mass; (m^2/s^2)
$K_H = \dfrac{\int_0^{\delta_T} v_r (T - T_\infty)\,\mathrm{d}z}{(T_w - T_\infty)\int_0^{\delta_T} v_r \mathrm{d}z}$	Shape-factor of the temperature profile; (–)
$K_m = \delta^{-1}\int_0^{\delta} \frac{v_r}{\omega r}\mathrm{d}z$	Non-dimensional radial mass flow rate through the boundary layer; (–)
$K_V = \dfrac{\int_0^{\infty} v_r (v_\varphi - v_{\varphi,\infty})\mathrm{d}z}{(\omega r - v_{\varphi,\infty})\int_0^{\infty} v_r \mathrm{d}z}$	Shape-factor of the velocity profile; (–)
L	Length of a single channel/pipe; (m)
L	Characteristic length in a two-pass channel; (m)
$M = -2\pi \int_0^b r^2 \tau_{w\varphi}\mathrm{d}r$	Moment of one side of a rotating disk; (Pa m^3)
$\dot{m}$	Total radial mass flowrate through the cavity between two rotating disks; (kg/s)
$\dot{m}$	Mass flowrate through a pipe/channel; (kg/s)
$\dot{m}_\mathrm{d} = 2\pi r\rho \int_0^{\delta} v_r \mathrm{d}z$	Mass flowrate through the momentum boundary layer over a rotating disk; (kg/s)
$\dot{m}_\mathrm{d,T} = 2\pi r\rho \int_0^{\delta_T} v_r \mathrm{d}z$	Mass flowrate through the thermal boundary layer over a rotating disk; (kg/s)

n	Exponent in the power-law approximation of the velocity profiles; (–)
n_{T}	Exponent in the power-law approximation of the temperature profiles; (–)
n_*	Exponent in the power-law approximation of the surface temperature, Eqs. (2.29)–(2.31); (–)
$N = \mathrm{v}_{r,\infty}/(\omega r)$	Non-dimensional radial velocity in potential flow outside of the boundary layer; (–)
$Nu = \frac{q_w r}{\lambda(T_w - T_\infty)}$	Local Nusselt number for a rotating disk; (–)
$Nu = \alpha D_{\mathrm{h}}/\lambda$	Local Nusselt number, single pipe/channel; (–)
$Nu = \alpha D_{\mathrm{e}}/\lambda$	Local Nusselt number, single pipe/channel; (–)
$Nu = \alpha D_{\mathrm{hi}}/\lambda$	Local Nusselt number in a two-pass channel; (–)
$\overline{Nu} = \bar{\alpha} D_{\mathrm{h}}/\lambda$	Average Nusselt number, single pipe/channel; (–)
$\overline{Nu} = \bar{\alpha} D_{\mathrm{e}}/\lambda$	Average Nusselt number, single pipe/channel; (–)
$\overline{Nu}_0$	Average Nusselt number, standard conditions (smooth straight pipe/channel, no rotation); (–)
$\overline{Nu} = \bar{\alpha} D_{\mathrm{hi}}/\lambda$	Average Nusselt number, two-pass channel; (–)
$\overline{Nu}_{\mathrm{st}}$	Average Nusselt number, straight smooth channel, CFD simulations; (–)
Nu_0	Nusselt number, Dittus-Boelter Eq. (7.10); (–)
Nu_1	Nusselt number, Eq. (8.2); (–)
$Nu_{\mathrm{b}} = \frac{q_w b}{\lambda(T_w - T_\infty)}$	Nusselt number based on the outer radius of a rotating disk; (–)
$Nu_D = \frac{q_w D}{\lambda(T_w - T_\infty)}$	Local Nusselt number based on the diameter of a rotating disk; (–)
$Nu_{Dj} = \frac{q_w D_j}{\lambda(T_w - T_\infty)}$	Local Nusselt number at flow impingement onto a rotating disk; (–)
$Nu_{\mathrm{av}} = \frac{q_{w.\mathrm{av}} b}{\lambda(T_w - T_\infty)_{\mathrm{av}}}$	Average Nusselt number for a rotating disk; (–)
$Nu_{\mathrm{av}} = \frac{b\int_0^b Nu(T_w - T_\infty)\mathrm{d}r}{\int_0^b (T_w - T_\infty) r\mathrm{d}r}$	Average Nusselt number for a rotating disk; (–)
p	Static pressure; (Pa)
p	Pitch between ribs; (m)
P_e	Perimeter of the pipe/channel; (m)
$Pr = \mu c_p/\lambda$	Prandtl number; (–)
q	Heat flux per unit area; (W/m^2)
$q_w = -\lambda\left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)_{z=0}$	Wall heat flux per unit area; (W/m^2)
$q_{w,\mathrm{av}} = \int_0^b q_w r\mathrm{d}r / \int_0^b r\mathrm{d}r$	Surface-averaged wall value of the heat flux per unit area; (W/m^2)
$Ra_H = \frac{\omega^2 H^3 D_{\mathrm{h}} \beta \Delta T}{2\lambda v}$	Rayleigh number for a rotating pipe; (–)
$Re_a = aD^2/\mathrm{v}$	Reynolds number in radial flow over a disk; (–)

$Re_\omega = \omega r^2/\nu$	Local rotational Reynolds number for a rotating disk; (–)
$Re_\omega = \omega D_h^2/\nu$	Rotational Reynolds number for a rotating pipe; (–)
$Re_{\omega D} = \omega D^2/\nu$	Rotational Reynolds number based on the disk diameter; (–)
$Re_{\omega j} = \omega D_j^2/\nu$	Rotational Reynolds number based on the nozzle diameter of an impinging jet; (–)
$Re_\Omega = \Omega r^2/\nu$	Local rotational Reynolds number for rotating cones in cone-disk systems; (–)
$Re_\varphi = \omega b^2/\nu$	Rotational Reynolds number at the outer radius of a disk; (–)
$Re_j = V_j D_j/\nu$	Reynolds number based on an impingement velocity; (–)
$Re_{V*} = V_* \delta/\nu$	Reynolds number based on the velocity V_*; (–)
$Re_T^{**} = \dfrac{\omega\, r \delta_T^{**}}{\nu}$	Enthalpy Reynolds number; (–)
$Re = Re_\Omega \eta_1^2/12,$	Reynolds number, cone-and-plate systems; (–)
$Re = Re_\omega \eta_1^2/12$	Reynolds number, cone-disk systems; (–)
$Re = \bar{V} D_\mathrm{h}/\nu$	Axial Reynolds number in a pipe; (–)
$Re = U_i D_\mathrm{hi}/\nu$	Axial Reynolds number, two-pass channel; (–)
$Ro = \omega D/\bar{V}$	Rossby number in a rotating pipe; (–)
r, φ, z	Cylindrical polar coordinates; (m or rad)
s	Spacing (height) between rotating disks; (m)
s	Thickness of a disk in the problem of unsteady conjugate heat transfer; (m)
S	Cross-section area of a pipe/channel; (m^2)
S	Contact surface; (m^2)
$Sc = \nu/D_m$	Schmidt number; (–)
$Sh = \alpha_m r/D_m$	Sherwood number for a rotating disk; (–)
$Sh_\mathrm{av} = \alpha_{m,\mathrm{av}} b/D_m$	Average Sherwood number for a rotating disk; (–)
$St = \dfrac{q_w}{\rho\, c_p V_* \,(T_w - T_\infty)}$	Stanton number; (–)
t	Time; (s)
T	Temperature; (K)
T_B	Bulk fluid temperature; (K)
T_i	Inlet temperature in a pipe/channel; (K)
T_w	Stationary/instantaneous wall temperature; (K)
$T_{w,i}$	Initial value of the wall temperature in unsteady heat transfer; (K)
T_∞	Temperature in potential flow outside of the boundary layer; (K)
T_ref	Reference temperature; (K)
$T_\mathrm{m-out} = \dfrac{1}{\dot{m}} \int_s T \mathrm{d}\dot{m}$	Temperature of mixing at the pipe outlet; (K)

$T^+ = (T_w - T)\rho_\infty V_\tau / q_w$	Local temperature in wall coordinates; (K)
$\tan\varphi = \dfrac{v_r}{\omega r - v_\varphi}$	Tangent of the flow swirl angle; (–)
$V = \left[v_r^2 + (v_\varphi - \omega r)^2\right]^{1/2}$	Total velocity; (m/s)
$V^+ = V/V_\tau$	Total velocity in wall coordinates; (m/s)
$V_\tau = (\tau_w/\rho)^{1/2}$	Friction velocity; (m/s)
v_r, v_φ, v_z	Velocity components in cylindrical coordinates; (m/s)
$\bar{v}_r = \dfrac{v_r}{\omega r - v_{\varphi,\infty}}$	Non-dimensional radial velocity; (–)
$\bar{v}_\varphi = \dfrac{v_\varphi - \omega r}{v_{\varphi,\infty} - \omega r}$	Non-dimensional tangential velocity; (–)
$V_* = \omega r \lvert\beta - 1\rvert (1 + \alpha^2)^{1/2}$	Characteristic velocity; (m/s)
V_j	Axial flow velocity at infinity or at the outlet of a nozzle; (m/s)
$\bar{V}$	Mean axial velocity in the pipe; (m/s)
u, v, w	Velocity components in Cartesian coordinates; (m/s)
U	Velocity in a two-pass channel; (m/s)
U_b	Bulk-averaged velocity, two-pass channel; (m/s)
U_i	Channel mean axial velocity at the inlet of a two-pass channel; (m/s)
W	Width of a rectangular channel; (m)
W_{in}	Width of the inlet pass, two-pass channel; (m)
W_{out}	Width of the outlet pass, two-pass channel; (m)
W_{el}	Tip wall distance from the divider wall; (m)
W_{web}	Divider wall thickness; (m)
x, y, z	Cartesian coordinates; (m)
$x = r/b$	Non-dimensional radial coordinate; (–)
$y = z/(0.5s)$	Non-dimensional axial coordinate in the problem of heat conduction inside a disk; (–)
$\tilde{z} = z/h$	Non-dimensional coordinate in conical gaps; (–)
$z^+ = zV_\tau/\nu$	Wall-law coordinate; (–)
$z^* = z/D_h$	Dimensionless axial coordinate in a pipe; (–)
$\alpha = -\tau_{wr}/\tau_{w\varphi}$	Tangent of the flow swirl angle on the wall; (–)
α	Heat transfer coefficient; (W/(m^2 K))
αS	Overall cooling efficiency; (W/K)
α_m	Mass transfer coefficient; (m/s)
$\alpha_{m,\text{av}} = \dfrac{2}{b^2}\displaystyle\int_0^b \alpha_m r\,\mathrm{d}r$	Surface-averaged mass transfer coefficient; (m/s)

$\beta = v_{\varphi,\infty}/(\omega r)$	Parameter of flow swirl, i.e. Dimensionless tangential velocity component in potential flow outside of the boundary layer; (–)
β	Angle of attack at the inlet in a rotating pipe; (°)
γ	Angle of conicity between a cone and a disk; (°)
δ	Thickness of a momentum boundary layer; (m)
$\bar{\delta} = \delta/b$	Non-dimensional thickness of a momentum boundary layer; (–)
δ_T	Thickness of a thermal boundary layer; (m)
$\Delta = \delta_T/\delta$	Relative thickness of a thermal/diffusion boundary layer; (–)
ε	Dissipation rate of k per unit mass: (m^2/s^3)
δ^*	Displacement thickness; (m)
δ^{**}	Momentum thickness; (m)
$\delta_T^{**} = \int_0^{\delta_T} \frac{v_r}{\omega r} \frac{T - T_\infty}{T_w - T_\infty} dz$	Enthalpy thickness; (m)
$\overline{\delta_T^{**}} = \delta_T^{**}/\delta$	Non-dimensional enthalpy thickness; (–)
$\Delta P^* = \Delta p/(0.5\rho U_i^2)$	Relative pressure drop; (–)
$\Delta T = T_w - T_\infty$	Temperature difference on a surface; (K)
$\Delta T_{av} = \frac{\int_0^b (T_w - T_\infty) r dr}{\int_0^b r dr}$	Surface-averaged temperature difference; (K)
$\Delta T_i = T_{w,i}(r) - T_\infty$	Surface temperature difference at the initial moment of time $t = 0$ in unsteady conditions; (K)
$\Delta T_t(t,r) = T_w(t,r) - T_\infty$	Instantaneous temperature difference on a surface in unsteady heat transfer; (K)
$\Delta T_{x=1}$	Temperature difference on a surface at $x = 1$; (K)
$\overline{\Delta T} = \Delta T/\Delta T_{x=1}$	Relative non-dimensional temperature difference on a surface; (–)
ζ	Self-similar variable, Eq. (2.26); (–)
$\Theta = \frac{T - T_w}{T_\infty - T_w}$	Non-dimensional temperature; (–)
$\theta = \frac{T - T_\infty}{T_w - T_\infty}$	Non-dimensional temperature; (–)
$\vartheta(t,r,z) = (T - T_\infty)/c_{0^*}$	Non-dimensional temperature inside a disk for unsteady heat transfer; (–)
$\kappa = \tan\varphi_\infty = \frac{v_{r,\infty}}{\omega r - v_{\varphi,\infty}}$	Tangent of the flow swirl angle, potential flow; (–)
λ	Thermal conductivity; (W/(m K))
μ	Dynamic viscosity; (Pa s)
ν	Kinematic viscosity; (m^2/s)
ν_T	Turbulent kinematic viscosity; (m^2/s)
$\xi = z/\delta$	Non-dimensional coordinate; (–)

$\xi_T = z/\delta_T$	Non-dimensional coordinate; (–)
ρ	Density; (kg/m^3)
τ	Shear stress; (Pa)
$\tau_w = (\tau_{wr}^2 + \tau_{w\varphi}^2)^{1/2}$	Total shear stress on the wall; (Pa)
$\tau_{wr} = \mu(dv_r/\mathrm{d}z)_{z=0}$	Radial shear stress on the wall; (Pa)
$\tau_{w\varphi} = \mu(dv_\varphi/\mathrm{d}z)_{z=0}$	Tangential shear stress on the wall; (Pa)
$\varphi_w = \arctan\left[\frac{v_r}{\omega\, r - v_\varphi}\right]_{z=0}$	Swirl angle at the wall of a rotating disk; (Pa)
$\varphi_w = \arctan\left[\frac{v_r}{\Omega\, r - v_\varphi}\right]_{z=0}$	Swirl angle at the wall of a stationary disk; (Pa)
χ	Reynolds analogy parameter, Eq. (2.52); (–)
ω	Angular velocity of rotation of a disk (or co-rotating disks); (1/s)
ω	Angular velocity of rotation of a pipe; (1/s)
ω	Specific dissipation rate of k (1/s)
Ω	Angular velocity of rotation of a fluid in rotating-disk systems; (1/s)
Ω	Angular velocity of rotation of a cone in cone-disk systems. (1/s)

Subscripts

av	Average value
c	Centrifugal forces (accelerations)
Cor	Coriolis forces (accelerations)
E	Ekman layers
i	Initial moment of time
i	Inlet to a cavity
in	Inlet to a channel
j	Impinging jet
lam	Laminar flow
max	Value at a point of maximum
ref	Reference value
t	Turbulent parameters
t	Transient/instantaneous value of a parameter
turb	Turbulent flow
T	Parameters of a thermal boundary layer
tr	Parameters at the point of abrupt transition from laminar to turbulent flow
tr1	Parameters at the point of the beginning of transition from laminar to turbulent flow
tr2	Parameters at the point of the end of transition from laminar to turbulent flow

tran	Transitional flow
w	Wall value (at $z = 0$)
w	Thermophysical properties of the wall material
0	Standard conditions
	(a) free rotating disk at $v_{r,\infty} = 0$ and $v_{\varphi,\infty} = 0$
	(b) smooth non-rotating pipe/channel of a circular cross-section
1	Boundary of the viscous/heat conduction layer
1	Outer cylindrical surface of a disk
2	Flat surface of a disk in unsteady heat transfer
∞	Potential flow outside of a boundary layer
$\overline{u'v'}$	Time-averaged pulsation turbulent values

Mathematical Symbols

$$\nabla^2 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}\right)$$ Operator Nabla

Acronyms

CFD	Computational fluid dynamics
DES	Detached-Eddy simulation
LES	Large-Eddy simulation
RANS	Reynolds-averaged Navier–Stokes (equations)
RSM	Reynolds stress model
TLC	Thermochromic liquid crystals
1D	One-dimensional
2D	Two-dimensional
3D	Three-dimensional