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New Approaches to Nonlinear Waves

Springer

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Preface

The theory of nonlinear waves is located right at the intersection of the linear wave theory, the theory of nonlinear oscillations, and the theory of nonlinear partial differential equations (PDEs), radiating into numerous fields of applied science, including studies in oceanography, nonlinear optics, plasma physics, weather and climate prediction.

It is hard to imagine that just a few decades ago the same equations arising in different fields of science were studied independently of each other, and the phenomena which they describe were called by different names. Probably one of the most illustrative examples of this kind is the phenomenon discovered in early 1960s, which is named modulational instability in purely mathematical texts, while known as modulation instability in nonlinear optics, as Benjamin-Feir instability in theory of water waves, as Oraevsky instability in plasma physics, etc. etc.

Only in the late 1980s gradually began to crystallize the idea of creating a new *nonlinear science* that would synthesize all known results into a single overall scheme and would allow to describe them all in one and the same language. I remember the annual meeting of the physical branch of the Russian Academy of Sciences, traditionally held at the Institute of Oceanology in Moscow, where Vladimir Eugenievich Zakharov very emotionally expounded the idea of creating such a language and writing an encyclopedia on nonlinear science, in which all of the most important results would be collected in one place. The late V.I. Arnold, who was in the audience, said that such language has long been there and it's called mathematics. Roars of laughter drowned out the answer of V.E. Zakharov.

Every physicist knows how long the road is from the physical level of accuracy, which is sufficient for solving many theoretical and practical problems in physics, to a rigorous mathematical definition and proof. The simplest example, as very often in physics, gives us the study of the physical pendulum. Galileo studied its movement and discovered the phenomenon of resonance in 1637. The rigorous mathematical definition of resonance was given by Poincaré 250 years later. And without an accurate notion of resonance, most of the chapters in this book principally could not have been written. In fact, much of the modern theory of nonlinear waves could not have been developed.

Since then many years have flown, and the notion of nonlinear science has become an integral part of our scientific language, and the first encyclopedia of nonlinear science saw the light of day in 2002 already, thanks to the invaluable effort of the late Alwyn Scott, who edited this great work of more than 1000 pages, written by scholars from all over the world.

However, the science does not halt, and new questions are coming forth, to which there is no answer in this encyclopedia. How to describe both discrete and kinetic regimes of wave turbulence resting on a unified strict mathematical approach? What new physical phenomena can be described if Phillips' definition of resonance is generalized to the case of moderate nonlinearity? Why does generalized NLS describe weakly nonlinear processes in water waves and strong nonlinear processes in optics? Etc. etc.

Answers to these and some other questions are given in this volume. A brief overview of individual chapters of the book is provided in the introductory Chap. 1. I also tried to position all the subjects in a logical consequence, i.e., scientific results, yielding new questions and then new results and again new questions *ad infinitum*. The choice of topics, of course, is biased and reflects my research interests and expertise. The last and longest Chap. 8 in the book has been written by Lev Shemer, one of the best modern experimentalists with water waves. The main goal of this chapter is to demonstrate why direct comparison of theoretical and numerical results with experimental measurements is often challenging; the author discusses in detail experiments devised to provide a basis for evaluation of the domain of validity of a theoretical model.

Understanding came to me already in 2011 that time is ripe again for gathering stones in the theory of nonlinear waves. At first, it was transformed into the idea of organizing a series of regular bi-annual conferences called *Wave Interaction* (WIN) to discuss new and promising topics in the area. The idea of writing a book was already discussed at *WIN*-2012. The format and content of the present volume was finalized during several meetings being held in 2012–2014:

- (2012) *Wave Interaction (WIN-2012)*, 23–26 April (Johannes Kepler University Linz, Austria)
- (2013) *Thematic Program on the Mathematics of Oceans*, April 29–June 28 (Fields Institute for Research in Mathematical Sciences, Toronto, Canada)
- (2014) *Weak Chaos and Weak Turbulence*, 3–7 February (Max Planck Institute for the Physics of Complex Systems, Dresden, Germany)
- (2014) *Wave Interaction (WIN-2014)*, 23–26 April (Johannes Kepler University Linz, Austria)
- (2014) *Theory of Water Waves*, 14 July–8 August (Isaac Newton Institute, Cambridge, UK)

All chapters are based on talks delivered at these conferences by selected invited speakers. I am very grateful to all attendants of these conferences who actively helped me to make a choice of topics. I would like to mention particularly N. Akhmediev, T. Bridges, W. Craig, A. Degasperis, K. Dysthe, R. Grimshaw,

P. Janssen, C.C. Mei, M. Onorato, D. Pelinovsky, E. Pelinovsky, A. Pikovsky, D. Shepelyansky, V. Shrira, and S.K. Turitsyn.

My aim was to create a book accessible to graduate students, engineers and researchers working in various fields of physics and applied mathematics. Consequently, the authors tried to make their exposition as clear as possible without harming scientific rigor. All theoretical chapters contain not only a conceptual background, but also illustrative examples of how these new techniques and approaches can be applied to specific problems. I am very much obliged to all the authors of this volume for their contributions and their patience when handling my remarks and making revisions.

I am also greatly indebted to all reviewers of the individual chapters which took over the hard and unremunerated work that resulted in tangible improvement of the quality of this book.

I am specially grateful to Shalva Amiranashvili, whose invaluable remarks and suggestions allowed me to improve the text of the introductory chapter.

I also would like to thank Dr. Aldo Rampioni and Kirsten Theunissen, Editors of the Springer Series Lecture Notes in Physics, who constantly assisted me during the preparation of the manuscript.

Linz, Austria Elena Tobisch
April 2015

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Acronyms

Here is the list of acronyms frequently used in this volume:

CHM	Charny-Hasegawa-Mima equation
IST	Inverse Scattering Transform
(g)NLS	(generalized) Nonlinear Schrödinger equation
(m)NLS	(modified) Nonlinear Schrödinger equation
(g)KdV	(generalized) Korteweg–de Vries equation
(m)KdV	(modified) Korteweg–de Vries equation
HAM	Homotopy Analysis Method
MI	Modulational Instability
ODE	Ordinary Differential Equation
PB	Peregrin Breather
PDE	Partial Differential Equation
RVP	Relaxed Variational Principle
SVEA	Slowly Varying Envelope Approximation
WTT	Weak (Wave) Turbulence Theory