Engineering Computation of Structures: The Finite Element Method

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Preface

The finite element method (FEM) has become an indispensable design tool in modelling and simulation of several engineering systems, and, therefore, the number of finite element computer programs that are now available to engineers is remarkable. All fields of engineering are facing great challenges in developing advanced systems that would contribute in solving problems affecting our life. Due to this fact, engineers must go through a very meticulous process of modelling, simulation, visualization, analysis, designing, prototyping, testing, and, finally, fabrication/construction. With modelling skills, the engineer gathers the physical and mathematical understanding of a problem, so the engineer combines with mathematics. Moreover, engineers with modelling and simulation skills must be able to analyze mathematically the finite element method.

This book provides unified and detailed course material on the FEM for engineers and students to solve primarily linear problems in mechanical engineering, with the main focus on static and dynamic structural problems. The purpose of the book is to contribute to the reader's understanding of the concepts, theories, and techniques used in the FEM. Fundamental and classical theories are introduced in a straightforward manner. Recent state-of-the-art treatment of engineering problems in designing and analyzing structural systems, including dynamic problems, is also discussed. Case studies are provided to implement the theory, the methodology, and techniques of the FEM. A text with significant concepts, theories, and modelling techniques, like the content in this book, helps engineers to use a commercial FEM software package in a professional and conscientious manner.

The FEM was originally developed to solve the stress field in continuum mechanics problems. Therefore, it seems appropriate to begin with the elasticity basic concepts and the classical theories of stressed materials and, only after, apply the relationship between forces, displacements, stresses, and strains on the process of modelling, simulating, and designing engineered technical systems. For this reason, the first chapter of this book is devoted to the mechanics of solids and structures by presenting the important basic principles. The derivation of key governing equations for 3D mathematical models is explained by means of drawings illustrating all the field variables and the relationships between them. Equations for various types of mathematical models, such as 2D and 1D models, are then obtained from the general equations for 3D mathematical models. From our teaching practices, the method of introducing the general 3D equations before approaching other structural components provides students a fundamental background that helps the process of learning the equations of additional mathematical models.

Practice without theory is blind, but theory without practice is sterile. Therefore, the readers of this book are encouraged to discover, within the finite element method, the proper relationship between theory and practice. During the last half century, a huge number of techniques have been developed in the area of the FEM, but very few of them are often used in engineering practice. This book does not want to be exhaustive in theories but be informative enough for the main useful techniques. All the finite element theories presented are applied explicitly in case studies. Most of them are presented sequentially, making it easier for readers to follow, and are discussed in a manner that clarifies concepts of the FEM.

A chapter-by-chapter description of the book is given below.

Chapter 1: Describes the important relationships associated with the elasticity basic concepts and the classical mathematical models for solids and structures. Important field variables of solid mechanics are introduced, and the dynamic equations of these variables are derived. Mathematical models for 2D and 3D solids, trusses, Euler beams, Timoshenko beams, frames, and plates are covered in a concise manner.

Chapter 2: The general finite element procedure is introduced and Hamilton's principle is used to establish the general forms of finite element equations. Concept of strong and weak solutions of a system equation of motion is discussed. Construction of shape functions for interpolation of field variables is described and their mathematical properties are also discussed. The finite element equations are discussed for static, eigenvalue analysis, as well as transient analyses.

Chapter 3: Describes the procedure used to obtain finite element matrices for truss structures. The procedures to obtain shape functions, the strain matrix, local and global coordinate systems, and the assembly of global finite element system equations are described. Very straightforward examples are used to demonstrate a complete and detailed finite element procedure to compute numerical solutions, emphasizing the differences between exact and numerical procedures.

Chapter 4: Extends the finite element procedure to the analysis of beam structures. Shape functions and finite element matrices are described for the Euler-Bernoulli beam. Examples to demonstrate the application of the finite element procedure into modal and transient analyzes are presented. Finite element matrices for frame structures are formulated by combining matrices of truss and beam elements. 3D beam formulation is described emphasizing the transformation of element matrices between the local and global coordinate systems. An example is given to demonstrate the use of 3D beams to solve practical engineering problems.

Chapter 5: Presents the finite element equations for the stress analysis of 2D structures under plane stress and plane strain conditions. Finite element matrices for 2D solids are derived, namely, the matrices of linear triangular elements. Concepts of isoparametric and superparametric representation are described and used to formulate linear and quadratic triangular finite elements. Linear and quadratic rectangular elements are derived in detail. Important considerations and requirements for the accuracy of the analysis results and the convergence of a numerical solution are introduced. An example of steel bracket is used to study the accuracy and convergence of different mathematical models selected: Euler-Bernoulli and Timoshenko beam theories and plane stress mathematical model.

Chapter 6: Describes the procedure used to obtain finite element matrices for plate and shell structures. Matrices for thin rectangular plate elements, which are based on Kirchhoff assumptions, are derived, and the continuity requirements for their shape functions are covered. Matrices for thick rectangular plate elements, following the Reissner-Mindlin plate theory, are derived in detail. Discussion on the boundary conditions used at both theories for modelling physical situations is presented. A flat shell element is formulated by combining a rectangular plate element and a rectangular 2D solid plane stress element. Meanwhile a general nine-node shell finite element is also formulated. The difference between flat and general shell elements is discussed on the Scordelis-Lo cylindrical roof benchmark problem, using the ADINA program.

Chapter 7: Finite element matrices for 3D solids are developed. Tetrahedron and hexahedron elements are formulated in detail and the volume coordinates are described within the process. Higher-order finite elements are also formulated, and an example of using 3D solids elements for a dental implant modelling is presented.

Chapter 8: Presents a discussion on some modelling techniques for the stress analyses of solids and structures. Mesh symmetry, rigid elements and constraint equations, mesh compatibility, modelling of offsets, supports, and connections between elements with different mathematical bases are all covered. Advanced modelling of laminated composite materials is also presented.

Most of the content in the book was selected from excellent existing books on the FEM (listed in the References), with a special contribution from the K-J Bathe books. This author has been making fundamental contributions to the finite element development and was the founder of ADINA R&D, Inc. which developed the program used for solving the practical examples presented at the book. The ADINA is also used in the practical lectures of two FEM courses taught by the authors in the University of Coimbra.

The mentor on the use of ADINA program for instruction in the University of Coimbra is, unfortunately, no longer with us, having passed away in 2010. Authors would like to pay tribute to Professor Nuno Ferreira Rilo for the excellent work he has done in spreading the teachings of FEM using this program.

Since FEM is well documented in many existing books, the information within this book was limited to the necessary minimum required for the use of commercial FEM software packages in a professional and conscientious manner. Readers seeking more advanced theoretical background are advised to refer to books such as those by K-J B.

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