


## One Hundred Prisoners and a Light Bulb

Hans van Ditmarsch • Barteld Kooi

# One Hundred Prisoners and a Light Bulb

*Illustrations by Elancheziyan*

 Springer

  
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Illustrations (c)2015 by Hans van Ditmarsch and Barteld Kooi

ISBN 978-3-319-16693-3

ISBN 978-3-319-16694-0 (eBook)

DOI 10.1007/978-3-319-16694-0

Library of Congress Control Number: 2015933648

Mathematics Subject Classification (2010): 00A08

Honderd gevangen en een gloeilamp  
Original Dutch edition published by (c) Epsilon Uitgaven, Amsterdam, 2013

Springer Cham Heidelberg New York Dordrecht London

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# Preface

This puzzlebook presents 11 different puzzles about knowledge and ignorance. Each puzzle is treated in depth in a separate chapter, and each chapter also contains additional puzzles for which the answers can be found at the back of the book. A constant theme in these puzzles is that the persons involved make announcements about what they know and do not know, and then later appear to contradict themselves. Such knowledge puzzles have played an important role in the development of an area known as dynamic epistemic logic. A separate stand-alone chapter gives an introduction to dynamic epistemic logic.

The illustrations for this book were made by Elanchezian. Elanchezian is a Tamil speaking Indian illustrator living in Chennai. Hans has an associate position at the Institute of Mathematical Sciences (IMSc) in Chennai, India. By the intermediation of his IMSc host Ramanujam, and the kind assistance of Shubashree Desikan, who acted as a Tamil-English interpreter, he got in contact with Elanchezian. How the illustrations to each chapter came about is story in itself, and we are very grateful for Elanchezian's essential part in this joint enterprise.

We wish to thank Paul Levrie and Vaishnavi Sundararajan for their substantial and very much appreciated efforts to proofread the final version of the manuscript. Peter van Emde Boas has indefatigably provided details on the history of the Consecutive Numbers riddle, and has much encouraged us in writing this book. We wish to thank Allen Mann, Springer, for his encouragement and for getting us started on this project. Nicolas Meyer from the ENS des Mines in Nancy found an embarrassing error in a light bulb protocol when Hans gave a course there, only a few weeks before we handed over the manuscript. He is one of many. If one were to go back all the 25 years of teaching logic and puzzles at colleges, universities, and summer schools, a much longer list of thanks to students and colleagues would be appropriate: by making an example of one, we wish to thank them all. No doubt, there will still be many remaining errors. They are all the responsibility of the authors.

Nancy, France, and Groningen,  
the Netherlands  
25 December 2014

Hans van Ditmarsch  
and Barteld Kooi

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