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Applied and Numerical Harmonic Analysis

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Compressed Sensing and its Applications

MATHEON Workshop 2013

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ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlyingmathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

Antenna theory	*Prediction theory*
Biomedical signal processing	*Radar applications*
Digital signal processing	*Sampling theory*
Fast algorithms	*Spectral estimation*
Gabor theory and applications	*Speech processing*
Image processing	*Time-frequency and*
Numerical partial differential equations	*time-scale analysis*
	Wavelet theory

The above point of view for the *ANHA* book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of "function." Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor's set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener's Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptivemodeling inherent in time-frequency-scalemethods such as wavelet theory.

The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'être of the *ANHA* series!

University of Maryland
College Park

John J. Benedetto
Series Editor

Preface

Compressed sensing is the idea that it should be possible to capture attributes of a signal using very few measurements. Since publication of the initial papers in 2006, it has captured the imagination of the international signal processing community, and the mathematical foundations are nowadays quite well understood. Key to compressed sensing is the surprising fact that high-dimensional signals, which allow a sparse representation by a suitable basis or, more generally, a frame, can be recovered from what were very few linear measurements by using efficient algorithms such as convex optimization approaches.

From the very beginning, the area profited from a fruitful interaction of applied mathematicians with engineers, with new applications leading to new mathematical methods and vice versa. Applications of compressed sensing to communication theory, imaging sciences, optics, radar technology, sensor networks, and tomography have been developed, with the technology in some areas more advanced than in others.

In December 2013, an international workshop was organized by the editors of this volume at the Technische Universität Berlin focusing specifically on application aspects of compressed sensing. In this sense, it was the first meeting with a focus on the application side of this novel methodology. The workshop was supported by the MATHEON, which is a research center in Berlin for "Mathematics for Key Technologies," as well as by the German Research Foundation (DFG). The workshop was attended by about 150 researchers from 13 different countries. Experts in a variety of research areas besides electrical engineering and mathematics were present among which were biologists, chemists, computer scientists, or material scientists. This mixture of people with different backgrounds oft led to particularly fruitful and inspiring discussions.

This book features contributions by three plenary speakers, namely Babak Hassibi (California Institute of Technology), Ali Pezeshki (Colorado State University), and Guillermo Shapiro (Duke University), and by thirteen invited speakers, namely Petros Boufounos (Mitsubishi Research Lab, USA), Volkan Cevher (EPFL), Shmuel Friedland (University of Illinois), Remi Gribonval (INRIA, Rennes), Anders Hansen (University of Cambridge), Peter Jung (Technische

Universität Berlin), Felix Krahmer (Technische Universität München, Germany), Dustin Mixon (Air Force Institute of Technology), Holger Rauhut (RWTH Aachen), Miguel Rodrigues (University College London), Rayan Saab (University of California, San Diego), Reinhold Schneider (Technische Universität Berlin, Germany), and Philipp Walk (Technische Universität München, Germany). It is the first monograph devoted to applications of compressed sensing. It is aimed at a broad readership including graduate students and researchers in the areas of mathematics, computer science, and engineering. However, it is also accessible to researchers working in any other field requiring methodologies for data science. Hence this volume can be used both as a state-of-the-art monograph on applications of compressed sensing and as a textbook for graduate students. Here is a brief outline of the contents of each chapter.

Chapter 1 is written by the editors and provides an introduction as well as a self-contained overview of the main results on the theory and applications of compressed sensing. It also serves to unify the notation throughout the whole book. Chapters 2–4 contain the contributions of the plenary speakers, whereas Chapters 5–15 feature the presentations of the invited speakers. Several chapters focus on problems one is facing when applying compressed sensing such as the problem of model mismatch (Chapter 3), quantization (Chapter 7), and unknown sparsifying dictionary (Chapter 8). Other chapters analyze specific constraints compressed sensing has to be adapted to for specific applications, in particular, structured sparsity, for which optimal sampling strategies (Chapter 5), algorithmic aspects (Chapters 4 + 12), and the specific structure of tensors (Chapters 9 + 14) are analyzed. Two chapters study theoretical obstacles, which, if overcome, would increase the impact of compressed sensing; Chapter 13 explores specific deterministic measurement matrices, and Chapter 11 explores co-sparsity-based reconstruction. The other chapters introduce and discuss the application of compressed sensing to areas such as acoustic imaging (Chapter 6), temporal color imaging (Chapter 2), and wireless communications (Chapters 10 + 15).

Finally, we would like to thank all members of the research group "Applied Functional Analysis" at Technische Universität Berlin, namely Martin Genzel, Mijail Guillemard, Anja Hedrich, Sandra Keiper, Anton Kolleck, Wang-Q Lim, Jackie Ma, Victoria Paternostro, Philipp Petersen, Friedrich Philipp, Rafael Reisenhofer, Martin Schäfer, Irena Bojarovska, and Yizhi Sun, without whom this MATHEON workshop would not have been possible.

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