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Quantum Communications

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To David, Shu-Ning, Gabriele, and Elena

Foreword

The birth of the original idea of Quantum Communications might be dated back to the same age when Claude E. Shannon formulated the mathematical theory of communications in 1948. In 1950, Dennis Gabor wrote a seminal paper on how to revise Information Theory by considering Quantum Physics, introducing the term “quantum noise”. Actually, as the carrier frequency goes up to few tens of a terahertz, quantum noise rapidly becomes more dominant than thermal noise. In 1960, Theodore H. Maiman succeeded in producing the first beam of laser light, whose frequency was at a few hundred terahertz. For a long time, it was a crucial trigger for full-scale studies on Quantum Communications. It was not, however, a straightforward task at all for researchers to establish the unification of the paradoxical aspects of Quantum Mechanics with the landmarks of Communications Theory. It was only recently that the core of Quantum Communications, that is, the theory of capacity for a lossy quantum-limited optical channel, was established. Until now, many new ideas and schemes have been added to the original standard scheme of Quantum Communications, represented by quantum key distribution, quantum teleportation, and so on. Realizing a new paradigm of Quantum Communications is now an endeavor in science and technology, because it requires a grand sum of not only the latest Quantum Communications technologies but also the basics of Information Theory and Signal Detection and Processing technologies. Therefore, it is not easy for students and researchers to learn all the necessary knowledge, to acquire techniques to design and implement the system, and to operate it in practice. These tasks usually take a long time through a variety of courses, and by reading many papers and several books.

This book is meant to achieve this very purpose. The author, Professor Gianfranco Cariolaro, has been working for a long time in the fields of Communications and Image Processing technologies, Deep Space Communications, and Quantum Communications. From this book, readers can track a history of Quantum Communications and learn its core concepts and very practical techniques. For this decade, commercial applications of quantum key distribution have been taking place, and in 2013, lunar laser communication was successfully demonstrated by the National Aeronautics and Space Administration, where a novel photon counting

method was employed. This means that an era of Quantum Communications in practice is around the corner. I am very excited to have this book at such a time. Through this book, readers will also be able to see a future Communications Technology on the shoulder of a long history of Quantum Communications.

Tokyo, Japan, September 2014

Masahide Sasaki
Director of Quantum ICT Laboratory
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and Communications Technology

Preface

Quantum Mechanics represents one of the most successful theories in the history of science. Born more than a hundred years ago, for several decades Quantum Mechanics was confined to a revolutionary interpretation of Physics and related fields, like Astronomy. Only in the last decades, after the discovery of laser with the possibility of producing coherent light, did Quantum Mechanics receive a strong interest in the area of information, with very innovative and promising applications (Quantum Computer, Quantum Cryptography, and Quantum Communications).

In particular, the original ideas of Quantum Communications were developed by Helstrom [10] and by scientists from MIT [11, 13] proving the superiority of quantum systems with respect to classic optical systems. However, the research in this specific field did not obtain the same spectacular expansion as the other fields of quantum information. In our personal opinion, the reason is twofold. One is the difficulty in the implementation of quantum receivers, which involves sophisticated optical operations. The other reason, perhaps the most relevant, was due to the advent of optical fibers, whose tremendous capacity annihilated the effort on the improvement of performances of the other transmission systems. This may explain the concentration of interest in the other fields of quantum information. Nevertheless, Quantum Communications deserve a more adequate attention for us to be prepared for the future developments, being confident that a strong progress in quantum optics will be surely achieved.

There is another motivation for considering Quantum Communications, especially for educational purposes in Information Engineering. In fact, continuing with our personal viewpoint, Quantum Mechanics is a discipline that cannot be ignored in the future curriculum of information engineers (electronics, computer science, telecommunications, and automatic control). On the other hand, Quantum Mechanics is a difficult discipline for its mathematical and also philosophical impact, and cannot be introduced at the level of Physics and Mathematical Physics because the study burden in information engineering is already quite heavy. However, we realized (with some surprise) that the notions of Quantum Mechanics

needed for Quantum Communications may be easily tackled by information engineering students. In fact, the notions needed at this level (vector spaces and probability theory) are already known to these students and require only an ad hoc recall. Following these ideas, six years ago the author introduced a course on Quantum Communications in the last year of the Telecommunications degree (master level) at the Faculty of Engineering of the University of Padova, and, as confirmed by students and colleagues, the conclusion was that the teaching experiment has proved very successful.

At the same time, experience shows that the majority of students, who join quantum optics and quantum information community after taking courses in quantum mechanics with concentration on elementary particles and high-energy physics, have very little feeling for the real notion of information transfer and manipulation as it is known in practical telecommunications. The comprehensive consideration of Quantum Communication concepts presented in this book serves to establish this missing conceptual link between the formal Quantum Mechanics theory formulated originally for particles and the quantum optical information manipulation utilizing quantum mechanics along with optics and telecommunications tools.

It is difficult to predict in what direction quantum information will evolve or when the quantum computer will arrive, but it will surely have a strong impact in the future. Students and researchers that will have learned Quantum Communications, having acquired the methodology and language, will be open to any other application in the field of Quantum Information.

Organization of the Book

The book is organized into three parts and 13 chapters.

Chapter 1 (Introduction) essentially describes the evolution of Quantum Mechanics in the previous century, with special emphasis on the last part of the evolution in the area of Quantum Information, with its promising and exciting applications.

Part I: Fundamentals

Chapter 2 collects the mathematical background needed in the formulation and development of Quantum Mechanics: mainly notions of linear vector spaces and Hilbert spaces, with special emphasis on the eigendecomposition of linear operators.

Chapter 3 introduces the fundamentals of Quantum Mechanics, in four postulates. Postulate 1 is concerned with the environment of Quantum Mechanics: a Hilbert space. Postulate 2 formulates the evolution of a quantum system, according to Schrödinger's and Heisenberg's visions. Postulate 3 is concerned with the quantum measurements, which prescribes the possibility of extracting information from a quantum system. Finally, Postulate 4 deals with the combination of two or more interacting quantum systems. A particular emphasis is given to Postulate 3,

because it manages the information in a quantum system and will be the basis of Quantum Communications and Quantum Information consideration.

Part II: Quantum Communications Systems

Chapter 4 deals with the general foundations of telecommunications systems and the difference between Classical and Quantum Communications systems. In the second part of the chapter the foundations of optical classical communications, which is the necessary prologue to optical quantum communications, are developed.

Chapter 5 develops the concept of optimal quantum decision, which establishes the best criterion to perform the measurements of Postulate 3 in a quantum system to extract information. Here a nontrivial effort is made to express the results within the language of telecommunications, where the quantum decision is applied to the receiver.

Chapter 6 develops suboptimization in quantum decision. Since optimization is very difficult, and exact solutions are only known in few cases, suboptimization techniques are considered, the most important of which is called square-root measurements (SRM).

Chapter 7 deals with the general formulation of quantum communication systems, where the transmitter (Alice) prepares and launches the information in a quantum channel and the receiver (Bob) extracts the information by applying the quantum decision rules. Although, in principle, the transmission of analog information would be possible, according to the lines of present-day technology, only digital information (data) is considered. In any case, we will refer to optical communications, in which the information is conveyed through a coherent radiation produced by a laser. The quantum formulation of coherent radiation is expressed according to the universal and celebrated Glauber's theory.

In the second part of the chapter, these basic ideas are applied to most popular quantum communication systems, each one characterized by a specific modulation format (OOK, PPM, PSK, and QAM). The performance of each specific system is compared to that of the corresponding classical optical system, where the decision is based on a simple photon counting. The comparisons will clearly state the superiority of the quantum systems.

Chapter 8 reconsiders the analysis of Chap. 7 with the introduction of thermal noise, in order to get a more realistic evaluation of the performance. Technically speaking, the analysis in the absence of thermal noise is carried out using the description of the system status made in terms of pure states, whereas the presence of thermal noise requires a description in terms of density operators. Consequently, the analysis becomes much more complicated (but challenging).

Chapter 9 deals with the implementation of coherent quantum communication systems. The few implementations available in the literature and the difficulties encountered in the realization are described. Also, some original ideas for an improved implementation of quantum communication systems are described.

Part III: Quantum Information

Chapter 10 begins by dealing with Quantum Information, which exhibits two forms, discrete and continuous. Discrete quantum information is based on discrete variables, the best known example of which is the quantum bit or qubit. Continuous quantum information is based on continuous variables, the best known example of which is provided by the quantized harmonic oscillator. An important remark is that most of the operations in quantum information processing can be carried out both with discrete and continuous variables (this last possibility is a quite recent discovery).

Chapter 11, Quantum Mechanics fundamentals of Chap. 3 are confined to the basic notions (relatively few) necessary to the development of Quantum Communications systems in Part II. In this chapter, for a full development of Quantum Information, the above fundamentals are extended to continuous quantum variables, to include Gaussian states and Gaussian transformations.

Chapter 12 deals with Information Theory, starting from Classical Shannon's Information Theory and then extending the concepts to Quantum Information Theory. The latter is a relatively new discipline, which is based on quantum mechanical principles and in particular on its intriguing resources, such as entanglement.

Chapter 13 deals with the applications of Quantum Information, as quantum random number generation, quantum key distribution, and teleportation. These applications are developed with both discrete and continuous variables.

Suggested Paths

For the choice of the path one should bear in mind that the book is a combination of Quantum Mechanics and Telecommunications, and perhaps students and researchers in the area of Information Engineering have no preliminary knowledge of Quantum Mechanics, whereas students and researchers in the area of Physics may have no preliminary knowledge of Telecommunications (for which we recommend reading Chap. 4 on Telecommunications fundamentals).

As said above, the mathematics needed for the comprehension of the book is confined to Linear Vector Spaces, as developed in Chap. 2. Hilbert spaces are introduced for completeness, but they are not really used. The other mathematical requirement is Probability Theory (probability fundamentals and random variables, sometimes extended to random processes). These preliminaries must be known at a good, but not too sophisticated level.

The book could be used by both graduate students (meaning people who have no knowledge of Quantum Mechanics) and researchers (meaning people who have a good knowledge of Quantum Mechanics, but not of classical Telecommunications) following two different paths.

In the Introduction we will indicate in detail two different paths for "students" and for "researchers".

Manuscript Preparation

To prepare the manuscript we used LATEX, supplemented with a personal library of macros. The illustrations too are composed with LATEX, sometimes with the help of Mathematica[®].

Padova, October 2014

Gianfranco Cariolaro

Acknowledgments

As an expert in traditional Telecommunications, in the twilight of my life, I decided to tread on the unknown territory of Quantum Information. This required the help of many people, without whom this book could never have been written. First of all, I would like to mention Tommaso Occhipinti and Federica Fongher, students, with whom I outlined the main ideas of the project.

I owe special thanks to Gianfranco Pierobon, who made fundamental contributions to many subjects in the book. In addition to sharing the same name, Gianfranco and I shared a similar enthusiastic wonder toward Quantum Mechanics, as we were both newcomers to the discipline. I still remember the trepidation and scepticism with which we submitted our first paper to an international journal (on the performance of Quantum Communications systems based on square root measurements). But it turned out to be a success. Incidentally, I would like to mention that the topic and the methodology of that paper were inspired by the work of Professor Masahide Sasaki and his collaborators. Gianfranco's help was so valuable that I repeatedly offered him to co-author the book, but he always refused, and, knowing his stubbornness, I had to give up. However, I hope he will accept next time, with the next book.

Roberto Corvaja was very helpful by re-reading the manuscript, over and over again, and integrating it with essential numeric computations. Nicola Laurenti provided invaluable assistance in the development of Part III of the book, concerning the applications of Quantum Information, in particular, by helping me to find my way in the jungle of this rapidly growing subject, as well as by proposing alternative arguments. Tomaso Erseghe was kind enough to learn Quantum Mechanics for the sole purpose of helping me, and he did so with great competence.

Several other dedicated readers offered me numerous, detailed, and insightful suggestions: Antonio Assalini, Luigi Bellato, Cesare Barbieri, Gianpaolo Naletto, Ezio Obetti, Stefano Olivares, Silvano Pupolin, Edi Ruffa, Lorenzo Sartoratti, Giovanna Sturaro, Francesco Ticozzi, Paolo Villoresi, and the young students: Nicola Dalla Pozza, Alberto Dall'Arche, Davide Marangon, and Giuseppe Vallone.

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I would like to mention that I was forced to interrupt my work for several months due to an unfortunate, tragic event that affected my family. On that occasion, three people in particular were crucial in helping me to cope with the situation and to return to normal life, namely Consul Vincenzo De Luca, Prof. Renato Scienza, and my friend Stefano Gastaldello. Other friends were very close to me in this difficult period, namely, Cesare Barbieri, Peter Kraniauskas, Umberto Mengali, Marina Munari, Silvano Pupolin, Romano Valussi, and Guido Vannucchi. I take the liberty to mention all this, an unusual subject for an acknowledgments section, because without the support of all these friends the book would have never been finished.

To all these people I owe a great debt of gratitude and offer heartfelt thanks.

Padova, October 2014

Gianfranco Cariolaro

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