Lecture Notes on Numerical Methods in Engineering and Sciences

Series editor

Eugenio Oñate, Barcelona, Spain

Editorial Board

Francisco Chinesta, Paris, France

Charbel Farhat, Stanford, USA

Carlos Felippa, Colorado, USA

Antonio Huerta, Barcelona, Spain

Thomas J.R. Hughes, Austin, USA

Sergio R. Idelsohn, Barcelona, Spain

Pierre Ladeveze, Paris, France

Wing Kam Liu, Evanston, USA

Xavier Oliver, Technical, Barcelona, Spain

Manolis Papadrakakis, Athens, Greece

Jacques Périaux, Jyväskylä, Finland

Bernhard Schrefler, Padova, Italy

Genki Yagawa, Tokyo, Japan

Mingwu Yuan, Beijing, China

Aims and Scope of the Series

This series publishes text books on topics of general interest in the field of computational engineering sciences.

The books will focus on subjects in which numerical methods play a fundamental role for solving problems in engineering and applied sciences. Advances in finite element, finite volume, finite differences, discrete and particle methods and their applications are examples of the topics covered by the series.

The main intended audience is the first year graduate student. Some books define the current state of a field to a highly specialised readership; others are accessible to final year undergraduates, but essentially the emphasis is on accessibility and clarity.

The books will be also useful for practising engineers and scientists interested in state of the art information on the theory and application of numerical methods.

More information about this series at <http://www.springer.com/series/8548>

Lev Baskin · Pekka Neittaanmäki
Boris Plamenevskii · Oleg Sarafanov

Resonant Tunneling

Quantum Waveguides of Variable
Cross-Section, Asymptotics, Numerics,
and Applications

Springer

Lev Baskin
Department of Mathematics
St. Petersburg State University
of Telecommunications
St. Petersburg
Russia

Pekka Neittaanmäki
Mathematical Information Technology
University of Jyväskylä
Jyväskylä
Finland

Boris Plamenevskii
Department of Mathematical Physics
St. Petersburg State University
St. Petersburg
Russia

Oleg Sarafanov
Department of Mathematical Physics
St. Petersburg State University
St. Petersburg
Russia

ISSN 1877-7341 ISSN 1877-735X (electronic)
Lecture Notes on Numerical Methods in Engineering and Sciences
ISBN 978-3-319-15104-5 ISBN 978-3-319-15105-2 (eBook)
DOI 10.1007/978-3-319-15105-2

Library of Congress Control Number: 2015935611

Springer Cham Heidelberg New York Dordrecht London
© Springer International Publishing Switzerland 2015

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media
(www.springer.com)

Preface

Devices based on the phenomenon of electron resonant tunneling are widely used in electronics. Efforts are directed toward refining properties of resonance structures. There are prospects for building new nanosize electronics elements based on quantum dot systems. However, the role of resonance structure can also be given to a quantum wire of variable cross-section. Instead of an “electrode—quantum dot—electrode” system, one can use a quantum wire with two narrows. A waveguide narrow is an effective potential barrier for longitudinal electron motion along a waveguide. The part of the waveguide between two narrows becomes a “resonator”, where electron resonant tunneling can occur. This phenomenon consists of the fact that, for an electron with energy E , the probability $T(E)$ to pass from one part of the waveguide to the other part through the resonator has a sharp peak at $E = E_{res}$, where E_{res} denotes a “resonant” energy. Such quantum resonators can find applications as elements of nanoelectronics devices and provide some advantages in regard to operation properties and production technology.

In the book, we study electron resonant tunneling in two- and three-dimensional quantum waveguides of variable cross-sections in the time-independent approach. We suggest mathematical models for resonant tunneling and develop asymptotic and numerical approaches for investigating the models. We also present schemes for several electronics devices based on the phenomenon of resonant tunneling. The book is addressed to mathematicians, physicists, and engineers interested in waveguide theory and its applications in electronics.

Contents

1	Introduction	1
1.1	Resonance Structures	1
1.2	Scattering Matrix	3
1.3	Method for Approximate Computation of Scattering Matrices	6
1.4	Asymptotic and Numerical Studies of Resonant Tunneling Resonant Tunneling in 2D Waveguides for Electrons of Small Energy	7
1.5	The Impact of a Finite Waveguide Work Function on Resonant Tunneling	11
1.6	Asymptotic Study of Resonant Tunneling in 3D Waveguides for Electrons of Small Energy	11
1.7	Electron Resonant Tunneling in the Presence of Magnetic Fields	12
1.8	Numerical Simulation of High Energy Electron Resonant Tunneling, the Fano Resonances	13
1.9	Asymptotic Analysis of Multichannel Resonant Tunneling	14
1.10	Electronics Devices Based on Resonant Tunneling in Waveguides of Variable Cross-Sections	14
2	Waveguides. Radiation Principle. Scattering Matrices	15
2.1	Boundary Value Problem in a Cylinder	17
2.1.1	Statement of the Problem. Operator Pencil	17
2.1.2	The Solvability of the Problem in a Cylinder	22
2.1.3	Asymptotics of Solutions	23
2.2	Problem in a Domain G with Cylindrical Ends	27
2.2.1	Statement and Fredholm Property of the Problem	27
2.2.2	Asymptotics of Solutions	28
2.2.3	Properties of the Index $\text{Ind } \mathcal{A}_\beta(\mu)$ and of the Spaces $\ker \mathcal{A}_\beta(\mu)$ and $\text{coker } \mathcal{A}_\beta(\mu)$	29
2.2.4	Calculation of the Coefficients in the Asymptotics	31

2.3	Waves and Scattering Matrices	35
2.3.1	Waves	35
2.3.2	Continuous Spectrum Eigenfunctions. The Scattering Matrix	37
2.3.3	The Intrinsic Radiation Principle	39
3	Properties of Scattering Matrices in a Vicinity of Thresholds	41
3.1	Augmented Space of Waves	42
3.1.1	Waves in a Cylinder	42
3.1.2	Waves in Domain G	45
3.2	Continuous Spectrum Eigenfunctions, Scattering Matrices	46
3.2.1	Intrinsic and Expanded Radiation Principles	46
3.2.2	Analyticity of Scattering Matrices with Respect to Spectral Parameter.	51
3.3	Other Properties of the Scattering Matrices	56
3.3.1	The Connection Between $S(\mu)$ and $S(\mu)$ for $\tau' < \mu < \tau$	56
3.3.2	The Connection Between $S(\mu)$ and $S(\mu)$ for $\tau < \mu < \tau''$	58
3.3.3	The Limits of $S(\mu)$ as $\mu \rightarrow \tau \pm 0$	62
4	Method for Computing Scattering Matrices	67
4.1	A Method for Computing Scattering Matrices Outside Thresholds	67
4.1.1	Statement of the Method	67
4.1.2	The Problem in G^R	69
4.1.3	Justification of the Method for Computing the Scattering Matrix	71
4.2	A Method for Computing Scattering Matrices in Vicinity of Thresholds	75
5	Asymptotic and Numerical Studies of Resonant Tunneling in 2D-Waveguides for Electrons of Small Energy	81
5.1	Statement of the Problem	81
5.2	Limit Problems	84
5.2.1	First Kind Limit Problems	84
5.2.2	Second Kind Limit Problems	86
5.3	Special Solutions to the First Kind Homogeneous Problems . . .	87
5.4	Asymptotic Formulas	90
5.4.1	Asymptotics of the Wave Function	91
5.4.2	Formulas for $\tilde{S}_{11}$, $\tilde{S}_{12}$, and C_{lj}	92
5.4.3	Formulas for Resonant Tunneling Characteristics	98

5.5	Justification of the Asymptotics	99
5.6	Comparison of Asymptotic and Numerical Results	110
5.6.1	Problems and Methods for Numerical Analysis.	110
5.6.2	Comparison of Asymptotic and Numerical Results . . .	114
5.7	The Impact of a Finite Waveguide Work Function on Resonant Tunneling	116
5.7.1	Preliminaries	116
5.7.2	A Qualitative Analysis of a Finite Work Function Impact on Electron Transport	119
5.7.3	Numerical Simulation of Resonant Tunneling with Regard to the Waveguide Work Function	121
6	Asymptotics of Resonant Tunneling in 3D Waveguides for Electrons of Small Energy	127
6.1	Statement of the Problem and Outline of the Results.	127
6.2	Limit Problems.	130
6.2.1	First Kind Limit Problems	130
6.2.2	Second Kind Limit Problems	132
6.3	Tunneling in a Waveguide with One Narrow	133
6.3.1	Special Solutions to the First Kind Homogeneous Problems.	134
6.3.2	Passing Through the Narrow	136
6.3.3	Formal Asymptotics	139
6.3.4	The Estimate of the Remainder.	142
6.4	Tunneling in a Waveguide with Two Narrows	145
6.4.1	Special Solutions to the Problem in the Resonator . . .	145
6.4.2	Formal Asymptotics	146
6.4.3	The Estimate of the Remainder.	154
7	Resonant Tunneling in 2D Waveguides in Magnetic Field.	163
7.1	Statement of the Problem.	163
7.2	The Limit Problems	166
7.2.1	First Kind Limit Problems	166
7.2.2	Second Kind Limit Problems	168
7.3	Special Solutions to Homogeneous First Kind Limit Problems.	169
7.4	Asymptotic Formulas	172
7.4.1	Asymptotics of the Wave Function	173
7.4.2	Formulas for $\tilde{S}_{11}$, $\tilde{S}_{12}$, and C_{lj}	174
7.4.3	Formulas for Resonant Tunneling Characteristics	178
7.5	Justification of the Asymptotics	179

7.6	Comparison of Asymptotic and Numerical Approaches	188
7.6.1	Problems and Methods for Numerical Analysis.	188
7.6.2	Comparison of Asymptotic and Numerical Results . . .	193
7.6.3	Dependence of Resonant Tunneling on the Magnetic Field Location in the Resonator	194
8	Effect of Magnetic Field on Resonant Tunneling in 3D Waveguides of Variable Cross-Section	197
8.1	Introduction	197
8.2	Statement of the Problem.	198
8.3	Limit Problems.	201
8.3.1	First Kind Limit Problems	201
8.3.2	Second Kind Limit Problems	203
8.4	Special Solutions of Limit Problems	205
8.5	Asymptotic Formulas	207
8.5.1	The Asymptotics of a Wave Function	207
8.5.2	Formulas for $\tilde{S}_{11}$, $\tilde{S}_{12}$, and $C_{11}, \dots, C_{14}$	209
8.5.3	Asymptotics for Resonant Tunneling Characteristics	212
8.6	Justification of the Asymptotics	214
9	Numerical Simulation of High Energy Electron Transport	223
9.1	Numerical Simulation of Multichannel Resonant Tunneling . .	223
9.1.1	Closed Resonator	223
9.1.2	The Method for Computing Scattering Matrix	224
9.1.3	Discussion of Numerical Results.	227
9.2	Fano Resonances	233
10	Asymptotic Analysis of Multichannel Resonant Tunneling	239
10.1	Statement of the Problem and Limit Problems	239
10.2	Tunneling in a Waveguide with One Narrow	242
10.2.1	Special Solutions to the First Kind Homogeneous Problems.	242
10.2.2	Asymptotic Formulas	244
10.3	Tunneling in a Waveguide with Two Narrows	248
10.3.1	Formal Asymptotics	248
10.3.2	The Estimate of the Remainder.	254

11	Electronics Devices Based on Resonant Tunneling	259
11.1	Magnetic Field Sensors Based on Quantum Waveguides	260
11.2	Transistors Based on Quantum Waveguides	264
11.3	Electron Flow Switch for Quantum Nets	267
	Bibliographical Sketch	271
	Bibliography	273