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Ingrid Hotz • Thomas Schultz
Editors

Visualization and Processing of Higher Order Descriptors for Multi-Valued Data

With 163 Figures, XXX in color

 Springer

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Preface

Multi-valued data arises in many applications ranging from modern imaging techniques to computational simulations and from medicine to engineering. Its mathematical modeling frequently involves higher-order descriptors, such as tensors or basis functions that are indexed by multiple indices. Even though such descriptors have a long tradition of being used in the sciences to describe physical phenomena and they are widely studied in mathematics, it is only over the last few years that their significance for data and image analysis and a need for their visualization has been recognized. At the same time, most applications that involve higher-order descriptors require collaborations between diverse scientific communities such as applied mathematics, computer science, physics, engineering, neuroscience, and medicine.

This book is the fifth in a series that fosters a more active exchange between these communities by collecting recent research findings and survey chapters dealing with different aspects of this topic authored by researchers from different fields. Most chapters were contributed by the participants of a workshop on the Visualization and Processing of Higher Order Descriptors for Multi-Valued Data that was held in February 2014 in Dagstuhl, Germany, and we hope that it will convey some of the workshop's inspiring atmosphere of open intellectual exchange. We are pleased to also include a number of high-quality chapters that are relevant to the topic, but whose authors could not attend the workshop due to other obligations or to the very limited number of available places.

Comparing the range of chapters to previous books in the series, we can observe a particularly strong interest in the mathematical foundations of the field. Similarly, statistical analysis of higher-order descriptors and the use of machine learning techniques play a more prominent role than in previous years. A renewed interest in diffusion-weighted magnetic resonance imaging (dMRI) can be attributed to the wider availability of multi-shell and multi-parameter data, as well as the emergence of a completely new generation of dMRI techniques that use more complex gradient waveforms. At the same time, applications in engineering and the physical sciences

continue to play an important role, and the workshop has witnessed promising collaborations in this domain.

The book is structured in five parts. The first is concerned with mathematical foundations, while the second brings together methods for the processing and interpolation of higher-order descriptors. The third part discusses questions of visualization. This is followed by a section on statistical analysis and a final part that presents solutions to specific application problems involving higher-order descriptors.

The **mathematical foundations** presented in Part I include the development of novel mathematical descriptions, the study and comparative analysis of existing ones as well as algorithms for their computation. A new generation of diffusion MRI techniques that make use of multiple gradient pulses or flexible gradient waveforms sparked a lot of interest at the workshop since they not only allow us to acquire information about tissue microstructure that is inaccessible to traditional diffusion MRI, but also produce data for which fully adequate models and visualization techniques are still to be developed. The first chapter unifies and compares two mathematical approaches to modeling this new type of data. The second chapter explores the use of Finsler geometry in the context of diffusion MRI by considering Brownian motion on Finsler manifolds. The third chapter relates the theory of orientation tensors and fabric tensors to parametric models of orientation distribution functions that are commonly used in dMRI. The fourth chapter studies the topology of linear symmetric tensor fields. Finally, the fifth chapter proposes two randomized algorithms for computing low-rank tensor approximations and applies them to an image-compression task.

The extension of data analysis methods from scalar fields to higher-order descriptors requires the generalization of the fundamental concepts of data **processing, filtering, and interpolation**. The first two chapters of Part II introduce morphological filters for higher-order descriptors. These are filters that are concerned with the detection and manipulation of shapes and structures in images with many applications. The topic of the first chapter is the extraction of long and thin structures, e.g. to find cracks. The second chapter proposes a partial ordering and a notion of maximum and minimum for color images using higher-order descriptors. The third chapter introduces a direction-controlled interpolation scheme to deal with tensor fields with conflicting orientations, e.g. to resolve fiber crossings in DTI fields. The last chapter summarizes the state of the art and challenges of tensor voting. The goal of tensor voting is to retrieve as much reliable information from various imaging data as possible, even where there is low resolution and in the presence of noise. The idea is to propagate local information encoded through tensors in a neighborhood following principles of proximity and similarity borrowed from Gestalt psychology.

The wealth of information present in higher-order descriptors poses significant challenges to **visualization**. The first three chapters in Part III address the visualization of data from different variants of diffusion MRI. The first presents

direct volume rendering and glyph-based strategies for data from diffusion spectrum imaging, which estimates the diffusion propagator, a three-dimensional probability distribution, at each point of three-dimensional space. The second chapter focuses on diffusion kurtosis imaging, which approximates the diffusion propagator in its covariance and kurtosis. It describes a method that uses this information to segment and visualize tissue types. Reconstructing the trajectories of major nerve-fiber bundles is a common goal in many variants of diffusion MRI, and it is significant in graphically conveying their shapes, the larger-scale bundles they form and their relationships to other anatomical structures in a clear manner. The third chapter surveys illustrative techniques that tackle these challenges. Finally, the extraction of features is a common strategy to deal with complex data. The last chapter compares different ways of computing the heat kernel signature, a popular tool in computational geometry, for general symmetric tensor fields and proposes its use as a feature for visualization.

Part IV is devoted to a more formal **statistical analysis** of higher-order descriptors, generalizing tools from univariate statistics. The first chapter presents a comprehensive framework for analyzing diffusion MRI data while accounting for multiple fiber compartments throughout the whole pipeline of interpolation, filtering, spatial normalization, and statistical analysis. The second chapter summarizes the state of the art and current challenges in applying statistical hypothesis testing and predictive modeling through supervised machine learning to multivariate neuroimaging data, again with a special focus on diffusion MRI.

The last part (Part V) is a collection of three chapters dealing with specific **applications** of higher-order descriptors. The first application is the analysis of a turbulent combustion simulation. A tensor-based clustering method is used to define typical and atypical behavior in the field. The second application is a classical mechanical engineering problem. It describes a case study for the use of tensorline visualization to support the design process for mechanical parts. The last chapter is concerned with a clinical application of diffusion-weighted magnetic resonance imaging (dMRI). It proposes enhancing local features taking context information into account. The framework developed is used for more accurate neurosurgical planning.

We would like to thank the organizers of the Dagstuhl workshop, Bernhard Burgeth (Universität des Saarlandes, DE), Ingrid Hotz (Linköping University–Norrköping, Sweden), Anna Vilanova Bartroli (TU Delft, NL), and Carl-Fredrik Westin (Harvard Medical School—Boston, US), as well as the board and staff of Schloss Dagstuhl for creating a unique opportunity for interdisciplinary exchange. We are also grateful to all the authors and reviewers who contributed to this book and who ensured its scientific quality. Last but not least, we would like to thank the editors of the Springer book series Mathematics and Visualization, as well as Martin Peters and Ruth Allewelt (Springer, Heidelberg) for their support in publishing this collection as part of their series.

We hope that this book will further the scientific progress on higher-order descriptors by serving as a reference to those who work with applications that generate multi-valued data or could benefit from higher-order models, and by providing a source of inspiration to researchers who are working on novel methods in the areas of image processing and visualization.

Norrköping, Sweden
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