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Beauville Surfaces and Groups

Springer

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Introduction

Ingrid Bauer, Shelly Garion and Alina Vdovina

"Beauville Surfaces" are certain rigid regular surfaces of general type, which can be described in a purely algebraic combinatorial way. Thus they play a very important role in different fields of mathematics such as algebraic geometry, group theory and number theory. The notion of Beauville surface was introduced by Fabrizio Catanese in 2000 and after the first systematic study of these surfaces by Ingrid Bauer, Fabrizio Catanese and Fritz Grunewald, there has been an increasing interest in the subject.

The conference "Beauville Surfaces and Groups 2012" was held in the University of Newcastle, UK, from 7 to 9 June 2012. This conference brought together, for the first time, experts from different fields of mathematics interested in Beauville surfaces, as well as young researchers and Ph.D. students, from the UK, USA, Germany, Italy and Spain, in order to share the status of the art and to discuss further developments in the study of these fascinating surfaces.

These conference proceedings include 11 chapters discussing Beauville surfaces (and generalizations of them), from different points of view, algebro-geometric and group-theoretic. Some of the chapters are expository survey papers and the others are research chapters on recent developments in this area. The chapters reflect the topics of the lectures presented during the workshop, and discuss various open problems and conjectures related to Beauville surfaces.

We briefly describe the content of the chapters.

The first chapter, by Ingrid Bauer, Fabrizio Catanese and Davide Frapporti, gives a comprehensive overview of what is currently known about the fundamental groups and, in particular, about the first homology groups of Beauville surfaces, and more generally, surfaces isogenous to a higher product of curves. The authors provide a computer script which calculates the first homology groups of regular surfaces of general type, hence in particular for Beauville surfaces.

The chapter by Nathan Barker, Nigel Boston, Norbert Peyemirnhoff and Alina Vdovina presents new interesting examples of finite groups admitting unmixed ramification structures giving rise to regular algebraic surfaces isogenous to a higher product of curves, generalizing Beauville surfaces. These groups are constructed as certain p-quotients of particular groups with special presentation related to finite projective planes and to expander graphs.

Nigel Boston overviews the important role played by a very interesting class of groups, namely p-groups, in the study of Beauville surfaces. The chapter describes recent results as to which p-groups admit a Beauville structure, with emphasis on ones of small order (due to Barker, Boston and Fairbairn) and ones that form inverse systems (due to Barker, Boston, Peyerimhoff and Vdovina).

Ben Fairbairn gives new interesting examples of finite groups which define real Beauville surfaces, and in particular, new examples of infinite families of groups admitting strongly real Beauville structures. The chapter discusses certain finite simple groups as well as abelian and nilpotent groups. It moreover deals with characteristically simple groups and almost simple groups.

Shelly Garion surveys the probabilistic group-theoretical approach towards proving well-known conjectures of Bauer, Catanese and Grunewald regarding Beauville surfaces arising from finite simple groups, by describing the following three works. The first is the work of Garion, Larsen and Lubotzky, showing that almost all finite simple groups of Lie type admit a Beauville structure. The second is the work of Garion and Penegini on Beauville structures of alternating groups, based on results of Liebeck and Shalev, and the third is the case of the group $\mathrm{PSL}_2(\mathrm{q})$, in which the author gives bounds on the probability of generating a Beauville structure.

The contribution of Christian Gleissner gives a complete classification of the connected components of the moduli space of surfaces of general type corresponding to regular unmixed surfaces S isogenous to a product of curves with $\chi(S) = 2$. These surfaces are natural generalizations of Beauville surfaces and the results require similar techniques from computational group theory. The surfaces studied in this research paper are no longer rigid, whence they come up in families, which are completely described.

Gareth Jones presents new interesting infinite families of Beauville surfaces arising from characteristically simple groups. More precisely, the author proves that if G is a cartesian power of a finite simple group of Lie type of low Lie rank or a sporadic simple group, then G is a Beauville group if and only if it has two generators and is not isomorphic to A_5. The author moreover conjectures that any cartesian power of a finite simple group of Lie type, which is 2-generated and not isomorphic to A_5, admits a Beauville structure.

The chapter by Kay Magaard and Chris Parker discusses the natural question of which extensions of the finite simple groups admit Beauville structures. This follows the previous exciting results due to Guralncik and Malle and to Fairbairn, Maggard and Parker, that all finite simple (respectively, quasi-simple) groups (except A_5 and $\mathrm{SL}_2(5)$) admit Beauville structures, thus proving a well-known conjecture of Bauer, Catanese and Grunewald. The chapter gives various interesting new examples of Frattini covers and semidirect products of finite simple groups admitting Beauville structures. As an application the authors deduce that every finite quotient of the profinite group $\mathrm{SL}_\mathrm{d}(\mathbb{Z}_\mathrm{p})$, where $d \geq 9$, admits a Beauville structure.

Matteo Penegini gives asymptotic estimates on the number of connected components of the moduli space of surfaces of general type, focusing on interesting families of surfaces isogenous to a higher product of curves, and Beauville surfaces in particular, and describes recent results due to Garion and Penegini. The proof is based on group theoretical methods, using the group theoretical terminology of the so-called "ramification structures", and Beauville structures in particular.

The paper by Roberto Pignatelli surveys the state of the art on quasi-étale quotients of a product of two curves, i.e. surfaces which are the quotient of a product of two curves by a finite group, acting freely outside a finite set of points. This class of surfaces has been successfully used in the last years to construct many interesting examples of surfaces of general type in a systematic computer aided way. The chapter reviews the principal results and gives a complete list of minimal quasi-étale surfaces of general type with geometric genus equal to the irregularity ≤ 2.

Francesco Polizzi gives a survey on recent work on isotrivially fibred surfaces and their numerical invariants. One of the main goals is, if $\alpha : X \to C$ is a relatively minimal isotrivial fibration of the surface X over a curve C, to relate the invariants K_X^2 and $\chi(\mathcal{O}_X)$ by some inequalities. Since surfaces X admitting an isotrivial fibration are of the form $X = C_1 \times C_2/G$, where $C_i, i = 1, 2$ are curves of respective genera at least two, and G is a finite group acting faithfully (but not necessarily freely) on $C_1 \times C_2$, these surfaces are natural generalizations of Beauville surfaces.

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These conference proceedings are dedicated to the memory of Fritz Grunewald*, who was the first to realize how fascinating Beauville surfaces are and inspired many mathematicians from different areas to investigate these surfaces. He is deeply missed.*