

Insight into Magnetorheological Shock Absorbers

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Foreword I

Direct transduction in hydraulic actuators (the production of useful output from a command signal without the intervention of a valving scheme), has been the dream of many hydraulic engineers who have struggled with the design of fast, stable, and reliable hydraulic valves. In the field of controllable suspension dampers, the design of responsive and well-behaved valves is an extremely difficult task due to the wide varying flow and pressure conditions within these dampers. This was the primary motivation for the development and use of both Magneto-Rheological (MR) and Electro-Rheological (ER) fluids in automotive semi-active suspension systems in the early 2000s (MagneRide™ by Delphi Automotive Systems, Corp/BWI Group) and in the early 2010s (eRRide™ by Fludicon GmbH), respectively.

The development of a novel technology has never been easy, and mastering a multidisciplinary one at a level sufficient for design of useful applications is even more demanding. Even though the controllable MR fluids were invented over 50 years ago, it was the more recent developments in the fields of MR fluids, suspension control algorithms, on-board power and processing electronics, and seal/bearing materials and design methods *plus* a clear understanding of the force and dynamic performance requirements of suspension dampers that enabled the first large-scale commercial use of MR fluids in a semi-active suspension system for passenger vehicles. So, the dream of a valveless damper was fulfilled about twelve years ago with astounding levels of performance, speed of response, dynamic authority, and reliability. As is usually the case with novel technologies, two additional generations have been developed and commercialized since the first generation of MagneRide™. The primary author and I worked closely on the development and application of the design and analysis methodology of this book. I can personally attest as to the suitability and utility of these techniques in the optimization of both static and dynamic performance of MR dampers.

This book covers comprehensively the fundamental science of MR fluids, their composition, and their performance characteristics. Relevant information is presented in a format that can be used in the design and optimization of MR damper pistons from the viewpoints of fluid flow and magnetics.

Basic architectures of MR dampers are presented along with alternative piston configurations, some of which have been commercialized successfully. Analytical methods that deal with the prediction of the piston flow and magnetic fields are included and then integrated into comprehensive analytical tools that deal with the complete damper, both from a static and a dynamic viewpoint. Detailed analyses of the flow and magnetic fields are presented by means of CFD and electromagnetic FE models. These models have proven extremely useful in clarifying some of the more subtle behaviors of MR dampers and in further optimizing the performance of MR dampers. In order to complete the “system,” the power electronics typically employed with MR dampers are presented in a clear fashion. The analytical models have been validated experimentally, and both static performance predictions and dynamic step responses are compared to laboratory test data.

For confidentiality reasons, the control algorithms (and sensors) used in the MagneRide™ system are not included. This is not a major omission because well-known algorithms (such as the “Skyhook” or those used in other commercial, valve-based semi-active systems) can be easily adapted to MR dampers.

A rather interesting emerging area of research and development is that of energy harvesting. Although the amount of power generated by typical road profiles (and converted into heat by the present hydraulic dampers) is rather small (of the order of 50–100 W) for vehicle propulsion, it is perhaps sufficient to drive internal sensors/controller electronics and power drivers, thus arriving at self-powered semi-active dampers. The present authors’ work on energy-harvesting actuators with MR fluids is included and should be valuable in promoting further research in this area.

This book is written by two authors with automotive damper engineering backgrounds as well as academic experiences in the field of Smart Materials. They have attempted to bring the MR technology closer to their readers. I believe that they have succeeded; this book should be particularly valuable to practicing engineers, researchers and students of MR fluids and their applications.

Darmstadt, Germany
17 November 2014

Alexander A. Alexandridis PhD
Chief Engineer (ret), Delphi Corporation
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MagneRide™ co-inventor

Foreword II

Technically, the words “magic” and “magnet” have unique derivations. Some etymologists suggest that the word “magic” predates “magnet” by roughly a century. From the 14th century the derivation of “magic” is generally “that which influences events and produces marvels using hidden natural forces”. So, it is not a stretch to suggest that for anyone that has anything from a casual passing interest to deep daily involvement with magneto-rheological fluids that “magic” is a most appropriate descriptor. In fact, I can confess that as the director at BWI Group with responsibilities for product and process engineering of the only serial production magneto-rheological dampers in the world, that I myself have answered more than once the question “how does magneto-rheological fluid work?” with the short phrase “it is magic!”

Of course, magneto-rheological fluid based dampers are technically not “magic”. They are, however, rather complex devices requiring a mastery of fluid dynamics, magnetic field theory, dynamic systems, electrical systems and more for one to truly comprehend their operation. In this book the authors expertly tackle all of these topics to decompose what appears to be “magic” into the broad basic physics which underpin these powerful devices. Beginning with magneto-rheological (MR) fluid basics the book combines both the theoretical and the empirical as it reveals details about dampers, control valves, modeling, damper configurations, and energy harvesting dampers. Adding to the usefulness for the reader the progression of topics in the book have been thoroughly and comprehensively referenced which easily enables further study of any of the major points.

So whether one has the aforementioned casual interest or works every day with MR based devices, this book will provide deeper understanding with just a selected

chapter or with a complete cover to cover read. Most certainly the book makes a valuable compilation of background, research, data, and prose which takes the “magic” which is MR dampers to better levels of understanding for any reader.

Dayton, Ohio, US
17 November 2014

Douglass L. Carson
Director—Suspension System Engineering
BWI Group

Preface

By definition, solids and fluids have been characterized with different physical laws. The behaviour of linear solids, for instance, can be quantified using Hooke's law of elasticity, and the rheology of linear fluids can be governed by Newton's viscosity law. Some materials, however, do not fit into the principal definition. Examples of such materials are magnetorheological/electrorheological (MR/ER) fluids well known under the name of smart fluids. The materials undergo major physical changes upon the application of an external (magnetic or electrical) stimulus so that they can be converted from a fluid to a pseudo-solid material. The reversible nature of the phenomena, the dramatic magnitude of changes and the speed of response have made them suitable for use in vibration isolation and control. The characteristics have been found useful in engineering systems where real-time performance, which follows changing conditions of system operation, is required.

No more than a scientific curiosity for decades since their discovery in the 1940s, both have deceived and tempted researchers and scientists for years. Till the early 1990s, the majority of research efforts concentrated on ER fluids; their limitations, electrical and safety issues, however, rendered them unsuitable for real-life applications at that time. MR fluids promptly stepped in, and once technological and control issues were overcome, the material was successfully utilized by the automotive industry in a valveless controlled chassis system application in passenger vehicle in the North American market in 2002. The system used MR fluid-based shock absorbers otherwise known as MR vehicle dampers. Since that time and as of 2012, the system marketed under the name of MagneRide has been put to a regular use in many passenger vehicle models as a standard suspension system or in the form of an option. A commercial application of a semi-active MR powertrain mount in a 2009 high performance passenger car completes the list.

Recently, one aspect of MR damper applications that has received a great deal of researcher's attention is energy harvesting. Here, in a typical configuration an MR device is driven by energy harvested from a vehicle while in motion. The mechanical energy that otherwise would be unused and lost through heat is converted into electricity and used for monitoring the output of an associated MR device. This trend is accompanied by the industry's interest in hybrid and electrical

vehicles. Vehicle suspension applications of energy harvesting dampers seem immediate; however, factors including the recovered energy magnitude, manufacturability, lifecycle, weight and cost may prevent such applications from commercialization in real-life.

Briefly, an automotive MR damper when used within a controlled environment, and in a vehicle suspension in particular, has its piston rod driven by a prescribed displacement/force. At the same time, the damper's coil is supplied with a current signal. The coil is located in the piston assembly of the damper. The commanded current is supplied to the coil through a pulse-width-modulated (PWM) driver. The current in the coil induces a magnetic field in the actuator in order to modify the MR fluid's yield stress and the damping force at the same time. The changes in the magnetic field passing through the components in the magnetic circuit of the actuator induce an electromotive force, whilst eddy currents are generated in the actuator's core. Furthermore, the eddy currents produce a magnetic field opposing the flux changes, and the speed of response of MR dampers becomes slower. Therefore, capturing the time-varying behaviour of the MR damper with the PWM current driver supplying the commanded current to the coil is necessary for describing the characteristics of an MR device. It involves a detailed description of the coil's resistance to the change of current and coupling among the magnetic field-induced yield stress and the damping force output (hydraulic circuit), as well as the dynamics of current drivers used for controlling the output of MR dampers.

In brief, the main objective of this book is to provide the readers with information on theoretical and practical aspects of MR damper operation, modelling and engineering. By definition, the word *insight* that is contained in the title of the book can describe a piece of information, an understanding of cause and effect based on identification of relationships and behaviours within a model or an instance of apprehending the true nature of a thing. By itself, it makes a promise to the potential readers and imposes obligations on the authors, and by following its spirit the authors hoped to provide the necessary foundations for the information in the book either in the form of theoretical knowledge or applied solutions. Specifically, the book contains the background information on smart fluids and related devices, common configurations as well as theory and its experimental verification. Following a review of the technology, theoretical backgrounds are provided of MR fluid compositions and key factors affecting the characteristics of these fluids is followed by a description of existing configurations of dampers and control valves. Specifically, the authors highlight common configurations of flow-mode MR dampers that have been considered by the automotive industry for controlled chassis applications. The authors focus on single-tube dampers utilizing a piston assembly with one coil or multiple coils and at least one annular flow channel in the piston.

Clearly, for modelling and design of a math-based analytical model of an exemplary MR fluid device, a flow-mode monotube damper is needed. Within the automotive industry it is a common practice to exercise entry-level scenarios with steady-state models, whereas more in-depth tests incorporating non-stationary and fluctuating magnetic fields are usually performed by means of more advanced tools

capable of copying the devices' dynamic features of interest. Here, the author's attention has been focused on models suitable for steady-state design analyses as well as dynamic studies, respectively. The task has been always somewhat of a challenge as with MR fluids the ability to model and quantify the behaviour of a material that is multidisciplinary by nature has always been a daunting exercise. It requires the knowledge of the material's rheology, electrical and mechanical engineering as well as control theory principles. First, a review of several constitutive models of non-Newtonian fluids in planar flow is carried out, and a novel exact (analytical) solution for them in terms of several non-dimensional parameters is obtained and analysed. The parameters capture the effects of plasticity, inertia, viscosity, shear thinning/thickening, and they allow for expressing the behaviour of an MR damper in pre-yield as well as post-yield flow regimes in a manner that is easy to follow and comprehend. Their application is highlighted in a fairly realistic steady-state model of a flow-mode MR damper configuration incorporating primary and secondary flow channels of MR fluid. A dynamic model of the damper for use in component-level as well as vehicle-level studies is also followed. In addition to copying the device's characteristics associated with the yield stress, the model also incorporates the effects of fluid compressibility, inertia, flow leakage past MR piston, friction, floating piston inertia, cavitation, gas pressure and the like. Furthermore, for the purpose of testing and verification, both models were applied to experimental data of several fabricated MR damper prototypes of a customized piston design and successfully verified across a wide range of piston velocity inputs, displacement amplitudes and coil current levels.

To summarize, Chap. 1 is an introduction to the material included in the book, whereas Chap. 2 includes a review of MR fluid formulations and key components affecting the fluids' performance in a semi-active environment. The information is followed in Chap. 3 by a thorough review of fundamental configurations of automotive flow-mode dampers. Next, in Chap. 4 the authors provide the reader with an application of several key non-Newtonian fluid models while in the MR damper environment. Chapter 5 contains a review of lumped parameter models of MR single-tube and double-tube dampers, respectively. Chapter 6 contains complementary information on MR fluid flow modelling using numerical Computational Fluid Dynamics (CFD) methods, whereas Chap. 7 reveals power driver structures for MR devices as well as control circuits and exemplary control strategies. In Chap. 8 the authors present results of several experiments involving customized prototypes of automotive flow-mode MR dampers. Finally, Chap. 9 highlights the development of energy harvesting MR dampers and Chap. 10 is a summary.

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Finally, we are tremendously honoured to have Douglass L. Carson, Director—Suspension System Engineering, BWI Group, and Dr. Alexander A. Alexandridis write the foreword to this book.

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About the Authors

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Nomenclature

A	Magnetic potential
A_H	Total area of holes
A_b	Bypass cross-section area
A_c	Coil window area
A_{core}	Core cross-section area
A_{eff}	Piston effective area
A_f	Flat plate area
A_g	Annulus cross-section area
A_m	Core active surface area
A_o	Orifice area
A_p	Cylinder (piston) cross-section area
A_r	Piston rod cross-section area
A_2	Check valve flow area
A_3	Check valve flow area
B	Magnetic flux density
B_{core}	Core flux density
B_g	Gap magnetic flux density
B_i	Iron magnetic flux density
B_s	Sleeve flux density
B_{sat}	Saturation flux density
$Bi = \frac{\tau_0 h}{(\mu \bar{v})}$	Bingham number
C	High velocity loss coefficient
C_H	Hole discharge coefficient
$C_f = \frac{\tau_w}{\rho \bar{v}^2}$	Dimensionless friction factor
c_f	Isothermal compressibility
C_o	Orifice discharge coefficient
C_2	Check valve discharge coefficient
C_3	Check valve discharge coefficient
D	Electric flux density
D_b	Bypass diameter

D_c	Piston core diameter
D_{dc}	EH damper core diameter
D_{dp}	EH damper piston diameter
D_{dp1}	EH damper kidney hole outer diameter
D_{dp2}	EH damper sleeve inner diameter
D_{dp3}	EH damper kidney hole inner diameter
D_{dr}	EH damper piston rod diameter
D_o	Orifice size
D_p	Piston outside diameter
D_r	Piston rod outside diameter
D_t	Cylinder inside diameter
D_2	Sleeve inside diameter or inner gap outer diameter (dual-gap MR valve)
D_3	Outer gap inside diameter
D_4	Outer gap outside diameter
d_{ci}	Generator coil window inside diameter
d_{co}	Generator coil window outside diameter
d_{mi}	Permanent magnet inner diameter
d_{mo}	Permanent magnet outer diameter
E	Electric field strength
E_s	Young modulus
e	Electromotive force
f	Frequency
F_a	Shear force
F_d	Damping force
F_{EH}	EH damper force
F_{fg}	Floating piston friction force
F_{fp}	Piston friction force
F_{fr}	Rod guide friction force
F_{min}	Minimum damping force
F_{max}	Maximum damping force
F_{off}	Off-state (min.) damping force
F_{on}	On-state (max.) damping force
F_1	Figure of merit (active volume)
F_2	Figure of merit (weight)
F_3	Figure of merit (power efficiency)
$G = -\frac{h\Delta p}{2L\tau_0}$	Dimensionless pressure number
G^*	Complex modulus
G_1	Inner gap pressure number
G_2	Outer gap pressure number
g_{ca}	Coil carcass thickness
g_h	Generator housing thickness
H	Magnetic field strength
H_c	Generator coil window height

H_{ca}	Generator coil carcass height
H_{dc}	EH damper coil window depth
H_g	Gap magnetic field strength
H_h	Generator housing height
h	Annulus height (gap size)
h_f	Flux bypass depth
h_1	Inner annulus height (gap size)
h_2	Outer annulus height (gap size)
H_{co}	Coil window height
H_g	Magnetic field strength in the annulus
$He = \frac{\tau_0 \rho h^2}{\mu^2}$	Hedstrom number
h_m	Generator permanent magnet height
I_{co}	Peak coil current
i_{co}	Coil current
I_{max}	Maximum coil current level
i_{cmd}	Coil current command
i_s	Steady-state current
i_{tr}	Transient current
j	Current density
$K_a = \frac{Q_a}{Q_p}$	Annular flow rate ratio
$K_b = \frac{Q_b}{Q_p}$	Bypass flow rate ratio
K_{co}	PI controller proportional setting
$K_f = \frac{F_{on}}{F_{off}}$	Force turn-up ratio
$K_l = \frac{Q_l}{Q_p}$	Leakage flow rate ratio
$K_Q = \frac{Q}{Q_p}$	Flow rate ratio
K_r	Proportional gain
$K_1 = \frac{Q_1}{Q_p}$	Inner gap flow rate ratio
$K_2 = \frac{Q_2}{Q_p}$	Outer gap flow rate ratio
k_c	Coil coupling coefficient
k_p	Average number of particles in unit area
k_α	Proportional gain of the controller output
k_β	Sensitivity of the current sensing
L	Annulus length
L_a	Active length
L_c	Compression chamber length
L_{ce}	Eddy current loop inductance
L_{co}	Coil inductance
L_{dp}	EH damper piston length
L_{dp2}	EH damper plate height
L_{dc}	EH damper active length
L_f	Complex transfer function amplitude

L_g	Gas chamber length
L_{go}	Generator coil inductance
L_r	Rebound chamber length
LDE	Life of device estimate
M_s	Magnetisation saturation
m	Flow index
m_g	Floating piston mass
m_r	Combined piston and rod mass
m_t	Tube mass
N_{co}	Coil wire turns
N_s	Number of chains per unit area
N_2	Search coil wire turns
n	Adiabatic constant
$P = -\frac{\Delta p}{L} \frac{wh^3}{12\mu Q}$	Dimensionless pressure (Philips 1969)
P_a	Atmospheric pressure
P_c	Compression chamber pressure
P_g	Gas chamber pressure
P_{g0}	Initial gas pressure
P_m	Mechanical power
P_r	Rebound chamber pressure
P_v	Vapour pressure
p	Pressure
p_{EH}	Instantaneous power
p_{out}	Outlet pressure
p_x	Pressure gradient
Q	Volumetric flow rate
Q_a	Volumetric flow rate through annulus
Q_b	Volumetric flow rate through bypass
Q_o	Volumetric flow rate through orifice
Q_p	Total volumetric flow rate (due to piston motion)
Q_0	Critical flow rate
Q_1	Inner gap flow rate
Q_2	Outer gap flow rate
Q_{v1}	Volumetric flow rate through MR valve
Q_{v2}	Volumetric flow rate through piston valve
Q_{v3}	Volumetric flow rate through base valve
r	Radial coordinate
R_{co}	Coil resistance
R_{c2}	Parasitic loop resistance
R_{ce}	Eddy current loop resistance
$Re = \frac{\rho \bar{v} h}{\mu}$	Reynolds number
Re_b	Bypass flow Reynolds number
R_{go}	Generator coil resistance
r	Particle radius

$S = \frac{12\mu Q}{wh^2\tau_0}$	Dimensionless plasticity number
S_0	Threshold plasticity
$T = \frac{wh^2\tau_0}{12\mu Q}$	Dimensionless yield stress (Philips 1969)
T_a	Damper temperature
T_{on}	PWM pulse duration
T_{off}	PWM pulse off-cycle time
T_i	Integral-acting factor of a PI controller
t	Time
t_c	Magnetic field establishment time
t_w	Tube thickness
U_{bat}	Battery (supply) voltage
U_{co}	Peak coil voltage
u_{co}	Coil voltage (across coil terminals)
u_{go}	Generator output voltage
U_{ref}	Reference voltage
u	Fluid velocity
u_z	Velocity gradient
u_1	Primary coil voltage
u_2	Search coil voltage
w	Annulus mean diameter
w_f	Flux bypass width
W_{co}	Coil window width
W_m^*	Mechanical power density
W_e^*	Electrical power density
w_1	Inner annulus mean diameter
w_2	Outer annulus mean diameter
V	Volume
V_c	Compression chamber volume
V_{c0}	Compression chamber initial volume
V_g	Gas chamber volume
V_{g0}	Gas chamber initial volume
v_g	Floating piston velocity
v_{in}	Inlet velocity
V_{min}	Minimum volume of MR fluid
V_p	Piston peak velocity
$V_{p,ref}$	Piston reference velocity
V_r	Rebound chamber volume
V_{r0}	Rebound chamber initial volume
V_s	Average volume of solid particles
v_p	Piston velocity
v_r	Piston rod velocity
v_t	Cylinder tube velocity
\bar{v}	Mean velocity

X_p	Piston displacement amplitude
x_g	Floating piston displacement
x_p	Piston displacement
x_r	Piston rod displacement
x_t	Cylinder tube displacement
Z_{co}	Damper impedance
z	Vertical coordinate
α	Fluid's plug width
α_d	Duty cycle
α_0	Constant
β	Bulk modulus
β_c	Container bulk modulus
β_f	Fluid bulk modulus
β_0	Pure fluid bulk modulus
$\gamma = \frac{\mu}{\mu_r}$	Dimensionless viscosity ratio
γ_d	PWM driver coefficient
γ_e	Material deformation
Δp	Pressure difference
Δp_a	Pressure difference across annulus
Δp_b	Pressure difference across bypass
Δp_H	Pressure difference across holes
Δp_{min}	Minimum pressure difference
Δp_{max}	Maximum difference across bypass
Δp_o	Pressure difference across orifice
Δp_1	Yielding pressure difference (across inner annulus)
Δp_2	Yielding pressure difference (across outer annulus)
$\delta = \frac{\tau_1}{\tau_2}$	Dimensionless yield stress ratio
δ_p	Gap between neighbouring particles
ε	Signal error estimate
μ	Fluid viscosity
μ_{app}	Apparent viscosity
μ_b	Base (carrier) oil viscosity
μ_m	Magnetic permeability
μ_{MR}	MR fluid permeability
μ_r	Pre-yield viscosity
μ_0	Free-space permeability
ν	Poisson coefficient
κ	Relative air content in MR fluid
κ_{EH}	Directional coefficient
κ_v	Susceptibility
κ_d	PWM driver coefficient
λ_{co}	Flux linkage
λ_0	Flux linkage initial condition
ρ	MR fluid density

σ_{EH}	Shift coefficient
τ	Shear stress
τ_{co}	Time constant
τ_w	(Wall) shear stress
τ_0	Static yield stress
τ_1	Static yield stress (bi-plastic Bingham model only)
τ_2	Dynamic yield stress (bi-plastic Bingham model only)
Φ	Magnetic flux
ϕ	Magnetic flux
θ_a	Angle between the centerline of the particle chain and the magnetic field direction
ϕ_g	Gap magnetic flux
ϕ_s	Steel core magnetic flux
ϕ_v	Solid phase volume fraction
ω	Angular velocity