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Zhiyong Chen · Jie Huang

# Stabilization and Regulation of Nonlinear Systems

A Robust and Adaptive Approach

 Springer

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*To Selina, Ruth, and Andrew*

Z. Chen

*To Qingwei, Anne, and Jane*

J. Huang

# Series Editors' Foreword

The topics of control engineering and signal processing continue to flourish and develop. In common with general scientific investigation, new ideas, concepts, and interpretations emerge quite spontaneously and these are then discussed, used, discarded, or subsumed into the prevailing subject paradigm. Sometimes, these innovative concepts coalesce into a new subdiscipline within the broad subject tapestry of control and signal processing. This preliminary battle between old and new usually takes place at conferences, through the Internet and in the journals of the discipline. After a little more maturity has been acquired by the new concepts then archival publication as a scientific or engineering monograph may occur.

A new concept in control and signal processing is known to have arrived when sufficient material has evolved for the topic to be taught as a specialized tutorial workshop or as a course to undergraduate, graduate, or industrial engineers. *Advanced Textbooks in Control and Signal Processing* are designed as a vehicle for the systematic presentation of course material for both popular and innovative topics in the discipline. It is hoped that prospective authors will welcome the opportunity to publish a structured and systematic presentation of some of the newer emerging control and signal processing technologies in the textbook series.

The Editors of *Advanced Textbooks in Control and Signal Processing* are aiming for the series to develop as a library of textbooks of high quality that cover some fundamental topics in the discipline but also include some coherent and systematic presentations of advanced topics that are gaining textbook status. A thorough and systematic presentation of the methods of nonlinear control has been one sought-after entry that has been missing from the series until now. The Editors are therefore understandably pleased to welcome this control textbook *Stabilization and Regulation of Nonlinear Systems: A Robust and Adaptive Approach* by authors Zhiyong Chen and Jie Huang into the series.

Practising control engineers deal with system nonlinearities by using a set of techniques that are extensions from the linear control paradigm. Methods like gain scheduling, controller scheduling, linear controller design across multiple models, and extended Kalman filters are all “constructive” approaches to nonlinear system problems. They have advantages based on simplicity of understanding and

implementation. But the field of nonlinear systems theory is now well developed and there is scope for more application of its techniques on industrial processes by control engineers. This course textbook for nonlinear systems methods adopts a structured approach to the theory that should assist engineers to understand the fundamentals, the subtleties, and the application of robust and adaptive approaches to nonlinear systems.

The authors are highly respected for their work in the nonlinear systems field and have interleaved the theory presented in the textbook with instructive demonstration examples. Zhiyong Chen is currently an Associate Professor of the School of Electrical Engineering and Computer Science at the University of Newcastle, NSW, Australia and has been awarded several best paper prizes at prestigious control conferences. Jie Huang is the Chairman and Choh-Ming Li Professor of Mechanical and Automation Engineering at the Chinese University of Hong Kong, Hong Kong, China. He has been a Distinguished Lecturer for the IEEE Control Systems Society, and is an IFAC and IEEE Fellow.

The book of Zhiyong Chen and Jie Huang can be described as a comprehensive and systematic course textbook on nonlinear control systems and will be welcomed by readers seeking to learn about and understand the foundations of the subject. However, the *Advanced Textbooks in Control and Signal Processing* series contains three titles that pursue nonlinear control, robust control, and adaptive control ideas in the context of specific application fields. Readers may also find the titles below of interest:

- *Analysis and Control of Nonlinear Process Systems* by Katalin M. Hangos, József Bokor and Gábor Szederkényi (ISBN 978-1-85233-600-4, 2003);
- *Robust and Adaptive Control with Aerospace Applications* by Eugene Lavretsky and Kevin Wise (ISBN 978-1-4471-4395-6, 2013); and
- *Robust Control Design with MATLAB<sup>®</sup>* by Da-Wei Gu, Petko Hr. Petrov and Mihail M. Konstantinov (ISBN 978-1-4471-4681-0, Second Edition, 2005).

Glasgow, UK, May 2014

M.J. Grimble  
M.A. Johnson

# Preface

Stabilization and output regulation are two central design problems in nonlinear control theory and applications. Stabilization refers to the process of designing a feedback control law for a controlled plant such that certain invariant manifold of the closed-loop system is stable in a broad sense. A motion trajectory and an equilibrium point of a dynamic system are special cases of an invariant manifold of the dynamic system. Output regulation, on the other hand, aims to design a feedback control law for a controlled plant subject to some exogenous signal such that certain invariant manifold of the closed-loop system is stable and, in addition, the output of the plant asymptotically approaches a given reference input determined by the exogenous signal. Thus, the output regulation problem is more demanding than the stabilization problem. Both the disturbance and the reference input of a control system can be viewed as the exogenous signal. Thus, the output regulation problem is also referred to as asymptotic tracking and disturbance rejection problem, or servomechanism design problem. In comparison with some other methods dealing with asymptotic tracking and/or disturbance rejection problem, a celebrated feature of the output regulation problem is that the control law is required to be able to handle a class of exogenous signals generated by a dynamic system called exosystem.

The output regulation problem arises from mathematically formulating practical control problems such as vibration suppression of high speed trains, disturbance rejection for flight vehicles, landing of aircraft on carriers under severe weather condition, and coordination and manipulation of robots. Thus this problem has attracted the attention of the control community for several decades and it has also been a driving force for the advancement of modern control theory and applications.

The output regulation problem was first studied for the class of linear time-invariant systems by Davison, Francis, and Wonham, to name just a few. The main tool for dealing with the output regulation problem is the internal model principle. By this principle, the output regulation problem of a given plant can be converted into the stabilization problem of the augmented system composed of the given plant and a well-defined dynamic compensator called internal model. The well-known



integral control can be viewed as a special case of the internal model principle when the exogenous signals are constant.

Since the early 1990s, the research on the output regulation problem has been focused on nonlinear systems. The problem has attracted the attention of numerous researchers from the world. By the time of the mid-2000s, the research on the nonlinear output regulation problem had achieved a degree of maturity. In addition to numerous research papers, four monographs on this topic were published.

- [1] C. I. Byrnes, F. Delli Priscoli, and A. Isidori, *Output Regulation of Uncertain Nonlinear Systems, Systems and Control: Foundations and Applications*, Birkhauser: Cambridge, 1997.
- [2] A. Isidori, L. Marconi, and A. Serrani, *Robust Autonomous Guidance: An Internal Model-Based Approach. Advances in Industrial Control*, Springer Verlag: London, 2003.
- [3] J. Huang, *Nonlinear Output Regulation: Theory and Applications*, SIAM: Philadelphia, 2004.
- [4] A. Pavlov, N. van de Wouw, and H. Nijmeijer, *Uniform Output Regulation of Nonlinear Systems: A Convergent Dynamics Approach*, Birkhauser: Berlin, 2006.

The book [1] studied the local output regulation problem of nonlinear systems. The book [2] focused on semi-global output regulation for flight vehicles using the internal model approach. The book [3] further studied both local and global output regulation or nonlinear uncertain systems by robust control approach. The book [4] was based on Jacobian analysis of nonlinear systems that is mainly effective for regional output regulation.

It is noted that the main tool for studying the output regulation problem in the first three books [1–3] is the internal model design approach. An internal model is a dynamic compensator which together with the given plant constitutes a so-called augmented system. An internal model is conceived such that the stabilization solution of the augmented system leads to the solution of the output regulation problem of the original plant. Thus, this design framework has made a connection between the stabilization problem and the output regulation problem. In contrast, the book [4] studied the problem using a convergent dynamics approach. In particular, a system is designed to have the property that all its solutions “forget” their initial conditions and converge to each other. The convergent steady-state solution is uniquely defined by an exosystem. Jacobian analysis of nonlinear systems was applied to achieve the convergent property.

Since the mid-2000s, the scope of research on the output regulation problem has experienced a tremendous expansion along several directions summarized as follows. First, several new methods for constructing more general internal models have been developed. These new internal models have led to the solution of the output regulation problem for more general nonlinear plants and exosystems on one hand, and, on the other hand, have generated several new types of robust stabilization problems for more complex nonlinear systems, thus leading to some interesting results and techniques for global stabilization of uncertain nonlinear

systems. Second, in order to handle uncertainty in exosystems, arbitrarily large exogenous signals and uncertain parameters, or unknown control direction, several adaptive control techniques have been incorporated into the original design framework for the output regulation problem, thus leading to the so-called adaptive output regulation problem. Third, extensive efforts have been made to solve real world practical control problems by output regulation theory. Some recent examples are asymptotic tracking of Chua's circuit, spacecraft attitude control, speed control of surface mounted motor, robust regulation of hyperchaotic Lozenz system, etc.

Given the rich new results of the nonlinear output regulation theory and applications obtained in recent years, and the need for studying the state-of-the-art techniques for handling the stabilization problem and the output regulation problem of nonlinear systems by graduate students and researchers in both academia and industries, we bring this book to readers. The book can be used as a textbook for graduate students in all engineering disciplines and applied mathematics. It can also be used as a reference book for both practitioners and theorists in broad areas of electrical engineering, aerospace engineering, mechanical engineering, and chemical engineering. Readers are assumed to have some knowledge of the fundamentals of linear algebra, advanced calculus, and linear systems.

The authors seek to strike a balance between the theoretical foundations of the output regulation problem and practical applications of the theory. The treatment is accompanied by many examples, including practical case studies with numerical simulations based on MATLAB. The book was typeset using L<sup>A</sup>T<sub>E</sub>X.

The development of this book would not have been possible without the support and help from many people. The authors are grateful to Alberto Isidori for his recommendation and encouragement for the publication of the book, and are indebted to the Series Editors Michael J. Grimble and Michael A. Johnson for their support and constructive comments. The Springer Editor Oliver Jackson, Managing Director Alexander Grossmann, and Senior Editorial Assistant Charlotte Cross are extremely helpful and enthusiastic in their advice and assistance. He Cai, Yi Dong, Wei Liu, and Maobin Lv, Ph.D. students of Jie Huang, Lijun Zhu and Haofei Meng, Ph.D. students of Zhiyong Chen, have proofread the manuscript. Some sections from Chaps. 6 to 9 are adapted from the joint publications with Lu Liu, Zhaowu Ping, Youfeng Su, Dabo Xu, and Xi Yang.

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Newcastle, Australia, April 2014  
Hong Kong

Zhiyong Chen  
Jie Huang

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# Symbols

$\ \cdot\ $	$\ x\ $	2-Norm of a vector $x$
$\ \cdot\ $	$\ A\ $	Induced 2-norm of a matrix $A$
$\mathbb{R}^n$	$x \in \mathbb{R}^n$	$n$ -Dimensional Euclidean space ( $\mathbb{R} = \mathbb{R}^1$ )
$\mathbb{R}^{n \times m}$	$A \in \mathbb{R}^{n \times m}$	Set of $n \times m$ matrices with elements in $\mathbb{R}$
$\mathbb{R}^+$	$x \in \mathbb{R}^+$	Non-negative real number set
$\mathbb{C}^n$	$x \in \mathbb{C}^n$	$n$ -Dimensional complex space ( $\mathbb{C} = \mathbb{C}^1$ )
$\mathbb{I}$	$x \in \mathbb{I}$	Set of integers
$I_n$ (or $I$ )	$\lambda I_n$ (or $\lambda I$ )	$n \times n$ identity matrix
$\in$	$\lambda \in \sigma(A)$	$\lambda$ is a member of $\sigma(A)$
$\notin$	$\lambda \notin \sigma(A)$	$\lambda$ is not a member of $\sigma(A)$
$\subset$	$A \subset B$	A set $A$ is a subset of a set $B$
$\otimes$	$A \otimes B$	Kronecker product
$\oplus$	$A \oplus B$	Tracy-Singh product
$\circ$	$f_1 \circ f_2$	Composition of functions $(f_1 \circ f_2)(x) = f_1(f_2(x))$
$\mapsto$	$f : \mathbb{A} \mapsto \mathbb{B}$	A function $f$ mapping a set $\mathbb{A}$ into a set $\mathbb{B}$
sup	$\sup_{t_1 \leq t \leq t_2} \ x(t)\ $	Supremum norm of $x(t)$ in $[t_1, t_2]$
col	$\text{col}(x_1, \dots, x_r)$	Vector stacked by $x_1 \in \mathbb{R}^{n_1}, \dots, x_r \in \mathbb{R}^{n_r}$
diag	$\text{diag}(A_1, \dots, A_r)$	Matrix diagonalized by $A_1 \in \mathbb{R}^{n_1 \times n_1}, \dots, A_r \in \mathbb{R}^{n_r \times n_r}$
T	$A^T$	Transpose of $A \in \mathbb{R}^{n \times m}$
lim sup	$\limsup_{k \rightarrow \infty} f(k)$	Limit superior
lim inf	$\liminf_{k \rightarrow \infty} f(k)$	Limit inferior

# Acronyms

AG	Asymptotic gain
AS	Asymptotically stable
ATDRP	Attitude tracking and disturbance rejection problem
ES	Exponentially stable
GARP	Global adaptive regulation problem
GAS	Globally asymptotically stable
GASP	Global adaptive stabilization problem
GES	Globally exponentially stable
GRORP	Global robust output regulation problem
GRSP	Global robust stabilization problem
GS	Globally stable
iISS	Integral input-to-state stable
ISS	Input-to-state stable
RAG	Robustly asymptotic gain
RES	Robustly exponentially stable
RGES	Robustly globally exponentially stable
RGS	Robustly globally stable
RISS	Robustly input-to-state stable
RORP	Robust output regulation problem
RUAS	Robustly uniformly asymptotically stable
RUGAS	Robustly uniformly globally asymptotically stable
SISO	Single input and single output
UAS	Uniformly asymptotically stable
UGAS	Uniformly globally asymptotically stable
US	Uniformly stable