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Decision Making and Optimization

Special Matrices and Their Applications in Economics and Management

Springer

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Preface

This book deals with properties of some special classes of matrices that are useful in economics and decision making problems.

In various fields of evaluation, selection, and prioritization processes decision maker(s) try to find the best alternative(s) from a feasible set of alternatives. In many cases, comparison of different alternatives according to their desirability in decision problems cannot be done using only a single criterion or one person. In many decision making problems, procedures have been established to combine opinions about alternatives related to different points of view. These procedures are often based on pairwise comparisons, in the sense that processes are linked to some degree of preference of one alternative over another. According to the nature of the information expressed by the decision maker, for every pair of alternatives different representation formats can be used to express preferences, e.g., multiplicative preference relations, additive preference relations, fuzzy preference relations, interval-valued preference relations, and also linguistic preference relations.

Many important optimization problems use objective functions and constraints that are characterized by extremal operators such as maximum, minimum, or various fuzzy triangular norms (t-norms). These problems can be solved using special classes of matrices, in which matrix operations are derived from the binary scalar operations of maximum and minimum instead of classical addition and multiplication. We shortly say that the computation is performed in max-min algebra. In a more general approach, the minimum operation, which itself is a specific t-norm, is substituted by any other t-norm T, and the computations are made in max-T algebra.

Matrices in max-min algebra, or in general max-T algebra, are useful in applications such as automata theory, design of switching circuits, logic of binary relations, medical diagnosis, Markov chains, social choice, models of organizations, information systems, political systems, and clustering. In these applications, the steady states of systems working in discrete time correspond to eigenvectors of matrices in max-min algebra (max-T algebra). The input data in real problems are usually not exact and can be rather characterized by interval values. For systems described by interval coefficients the investigation of steady states leads to computing various types of interval eigenvectors.

In Chap. 1 basic preliminary concepts and results are presented which will be used in the following chapters: t-norms and t-conorms, fuzzy sets, fuzzy relations, fuzzy numbers, triangular fuzzy numbers, fuzzy matrices, alo-groups, and others.

Chapter 2 deals with pairwise comparison matrices. In a multicriteria decision making context, a pairwise comparison matrix is a helpful tool to determine the weighted ranking on a set of alternatives or criteria. The entry of the matrix can assume different meanings: it can be a preference ratio (multiplicative case) or a preference difference (additive case), or, it belongs to the unit interval and measures the distance from the indifference that is expressed by 0.5 (fuzzy case). When comparing two elements, the decision maker assigns the value from a scale to any pair of alternatives representing the element of the pairwise preference matrix. Here, we investigate transitivity and consistency of preference matrices being understood differently with respect to the type of preference matrix. By various methods and from various types of preference matrices we obtain corresponding priority vectors for final ranking of alternatives. The obtained results are also applied to situations where some elements of the fuzzy preference matrix are missing. Finally, a unified framework for pairwise comparison matrices based on abelian linearly ordered groups is presented. Illustrative numerical examples are supplemented.

Chapter 3 is aimed on pairwise comparison matrices with fuzzy elements. Fuzzy elements of the pairwise comparison matrix are applied whenever the decision maker is not sure about the value of his/her evaluation of the relative importance of elements in question. We particularly deal with pairwise comparison matrices with fuzzy number components and investigate some properties of such matrices. In comparison with pairwise comparison matrices with crisp components investigated in the previous chapter, here we investigate pairwise comparison matrices with elements from alo-group over a real interval. Such an approach allows for generalization of additive, multiplicative, and fuzzy pairwise comparison matrices with fuzzy elements. Moreover, we deal with the problem of measuring the inconsistency of fuzzy pairwise comparison matrices by defining corresponding inconsistency indexes. Numerical examples are presented to illustrate the concepts and derived properties.

Chapter 4 considers the properties of equation/inequality optimization systems with max-min separable functions on one or both sides of the relations, as well as optimization problems under max-min separable equation/inequality constraints. For the optimization problems with one-sided max-min separable constraints an explicit solution formula is derived, a duality theory is developed, and some optimization problems on the set of points attainable by the functions occurring in the constraints are solved. Solution methods for some classes of optimization problems with two-sided equation and inequality constraints are proposed in the last part of the chapter.

In Chap. 5 the steady states of systems with imprecise input data in max-min algebra are investigated. Six possible types of an interval eigenvector of an interval matrix are introduced, using various combination of quantifiers in the definition. The previously known characterizations of the interval eigenvectors that were restricted to the increasing eigenvectors are extended here to the non-decreasing eigenvectors, and further to all possible interval eigenvectors of a given max-min matrix. Classification types of general interval eigenvectors are studied and characterization of all possible six types is presented. The relations between various types are shown by several examples.

Chapter 6 describes the structure of the eigenspace of a given fuzzy matrix in two specific max-T algebras: the so-called max-drast algebra, in which the least t-norm T (often called the drastic norm) is used, and max-Łukasiewicz algebra with Łukasiewicz t-norm L. For both of these max-T algebras the necessary and sufficient conditions are presented under which the monotone eigenspace (the set of all non-decreasing eigenvectors) of a given matrix is nonempty and, in the positive case, the structure of the monotone eigenspace is described. Using permutations of matrix rows and columns, the results are extended to the whole eigenspace.

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