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Extraction of Quantifiable Information from Complex Systems

Springer

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Foreword

In April 2007, the Deutsche Forschungsgemeinschaft (DFG) approved the Priority Program 1324 "Mathematical Methods for Extracting Quantifiable Information from Complex Systems". The objective of this volume is to offer a comprehensive overview of the scientific highlights obtained in the course of this priority program.

Mathematical models of complex systems are gaining rapidly increasing importance in driving fundamental developments in various fields such as science and engineering at large but also in new areas such as computational finance. Ever-increasing hardware capacities and computing power encourage and foster the development of more and more realistic models. On the other hand, the necessarily growing complexity of such models keeps posing serious and even bigger challenges to their numerical treatment.

Principal obstructions such as the *curse of dimensionality* suggest that a proper response to these challenges cannot be based solely on further increasing computing power. Instead, recent developments in mathematical sciences indicate that significant progress can only be achieved by contriving novel and much more powerful numerical solution strategies by systematically exploiting synergies and conceptual interconnections between the various relevant research areas. Needless to stress that this requires a deeper understanding of the mathematical foundations as well as exploring new and efficient algorithmic concepts. Fostering such well-balanced developments has been a central objective of this priority program.

The understanding and numerical treatment of spatially high-dimensional systems is clearly one of the most challenging tasks in applied mathematics. The problem of spatial high dimensionality is encountered in numerous application contexts such as machine learning, design of experiments, parameter-dependent models and their optimization, mathematical finance, PDEs in high-dimensional phase space, to name only a few, which already reflect the conceptual breadth. It is this seeming variability that makes a substantial impact of better exploiting conceptual and methodological synergies conceivable and in fact likely. It seems

that to be really successful, theoretical research and practical applications should go hand in hand. In fact, this volume reflects an attempt to realize a proper balance between research with a primary methodological focus and challenging concrete application areas, although these two regimes can, of course, not be strictly separated. To that end, it has appeared to be necessary to combine different fields of mathematics such as numerical analysis and computational stochastics. On the other hand, to keep the whole programme sufficiently focused, it seemed advisable to concentrate on specific but related fields of application that share some common characteristics that allow one to benefit from conceptual similarities.

On the methodological side, several important new numerical approximation methods have been developed and/or further investigated in the course of the priority program. First of all, as one of the central techniques, let us mention tensor approximations. New tensor formats have been developed, and efficient tensor approximation schemes for various applications, e.g. in quantum dynamics and computational finance, have been studied; see Chaps. 2, 10, 12, 16 and 19. Adaptive strategies with all their facets have been employed in most of the projects; see, e.g., Chaps. 2, 4, 5, 9, 10, 14 and 16. Closely related with adaptivity is of course the concept of sparsity/compressed sensing; see Chaps. 14 and 18. As further techniques, sparse grids (Chap. 9), ANOVA decompositions (Chap. 11) and Fourier methods (Chap. 17) have been investigated. As a quite new technique, the reduced basis methods also came into play (see Chap. 2), in particular in the second period of SPP 1324. Of course, tensor methods as well as model order reduction concepts such as the reduced basis method address spatially high-dimensional problems. Both paradigms use the separation of variables as the central means to reduce computational complexity. Moreover, they can be viewed as trying to exploit sparsity by determining specific problem- and solution-dependent dictionaries that are able to approximate the searched object by possibly few terms. Moreover, Chaps. 1, 6 and 20 are concerned with Monte Carlo and Multilevel Monte Carlo methods in the context of stochastic applications.

One of the major themes within SPP 1324 has been high-dimensional problems in physics. Chapter 21 is concerned with the regularity of the solution to the electronic Schrödinger equation. Chapter 19 studies problems in quantum dynamics, the chemical master equation is one of the topics in Chap. 15, and Chap. 11 is concerned with electronic structure problems. Another very important issue within SPP 1324 has been differential equations with random or parameter-dependent coefficients and their various applications. The theory and numerical treatment of these problems are discussed in Chaps. 2 and 7. Closely related with this topic are stochastic differential equations and stochastic partial differential equations. The adaptive numerical treatment of SPDEs is studied in Chap. 5. SDEs with their various applications such as stochastic filtering are discussed in Chaps. 1, 6 and 8. Additional fields of application have been computational finance (see Chap. 16) and inverse problems (see Chaps. 3 and 18).

Overall, the network of SPP 1324 comprised more than 60 scientists, and 20 projects were funded in two periods. Up to now, more than 170 papers have been published by the participants of SPP 1324. The aim of this volume is of course not

to give a complete presentation of all these results but rather to collect the scientific highlights in order to demonstrate the impact of SPP 1324 on further researches. The editors and authors hope that this volume will arouse interest in the reader in the various new mathematical concepts and numerical algorithms that have been developed in the priority program. For further information concerning SPP 1324, please visit http://www.dfg-spp1324.de/.

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