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Abdellah Benzaouia · Ahmed El Hajjaji

# Advanced Takagi–Sugeno Fuzzy Systems

## Delay and Saturation

Springer

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*To the memory of my father Mohamed and my mother Fatna*

Benzaouia

*To Ilyan, Lila and Fatima*

El Hajjaji

# Preface

This book presents a detailed background of Takagi–Sugeno fuzzy systems as a compilation of important results obtained by the authors during at least 10 years of research in the field of T–S fuzzy systems. The chapters are presented in chronological form to deal with the main problems studied by the authors. The first chapter focuses on the introduction of T–S representation for nonlinear systems together with the basic results on stability and stabilization for continuous-time and discrete-time systems with or without delay. Particular attention is paid to the way different T–S representations are obtained from a nonlinear system. Simple examples are studied to show the applicability of the T–S fuzzy method to nonlinear systems. This chapter is dedicated to readers not familiar with this tool as students and researchers. For the subsequent chapters, each chapter is devoted to a particular problem as:

- T–S fuzzy systems with input saturation,
- Stabilization of T–S fuzzy systems by output feedback,
- Positive T–S fuzzy systems,
- T–S fuzzy systems with varying time delays,
- Uncertain T–S fuzzy systems,
- Observers for T–S fuzzy systems,
- T–S fuzzy systems with both saturation and multiple delays using linear programing,
- Two-dimensional T–S fuzzy systems.

Most the studied problems have first been solved by the authors and their teams as T–S fuzzy systems with saturation, positive T–S fuzzy systems, stabilization conditions under linear programing, two-dimensional T–S fuzzy systems. All the presented results are illustrated by examples, generally real plant models and figures. Hence, the book contains about 70 figures and compiles 220 references. All the results are presented with their proofs and the references where they appeared for the first time.

The objective of the authors is to complete the available literature on T–S fuzzy systems with additional solved problems like saturation, positivity, and two-dimensional systems. The book is addressed to a large audience familiar or not with this topic.

Abdellah Benzaouia
Ahmed El Hajjaji

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Ahmed El Hajjaji

# Contents

# Acronyms

| | |
|---|---|
| 2D | Two dimensional |
| 2DC | Two dimensional continuous |
| 2DD | Two dimensional discrete |
| BMI | Bilinear matrix inequality |
| CFS | Continuous fuzzy system |
| co{.} | Convex hull of {.} |
| CTC | Continuous-time case |
| CTS | Continuous-time system |
| DFS | Discrete Fuzzy System |
| DTC | Discrete-time case |
| DTS | Discrete-time system |
| Eq. | Equation |
| Eqs. | Equations |
| LKF | Lyapunov–Krasovskii Functional |
| LMI | Linear matrix inequality |
| LP | Linear programming |
| LPV | Linear parameter variable |
| MIMO | Multiple input multiple output |
| MLF | Multiple Lyapunov function |
| NS | Non symmetrical |
| OPDC | Output parallel distributed compensation |
| PDC | Parallel distributed compensation |
| PLMI | Parameterized Linear Matrix Inequality |
| resp. | Respectively |
| s.t. | Such that |
| SISO | Single input single output |
| SOFC | Static output feedback control |
| T–S | Takagi–Sugeno |
| w.r.t. | With respect to |

# Notations

- If $x, y$ are vectors of $\mathbb{R}^n$, then $x \leq y$ stands componentwise.
- For a matrix $A \in \mathbb{R}^{n\times m}$, $|A|$ is the matrix formed by the absolute value of the components of $A$, while $\sigma(A)$ denotes its spectrum.
- For a vector $v_i \in \mathbb{R}^n, v_l^i$ indicates the *lth* component of the vector.
- *int* $\mathscr{D}$ denotes the interior of the set $\mathscr{D}$.
- For a square matrix $Q > 0, (Q \geq 0)$ if $Q \in \mathbb{R}^{n\times n}$ is positive definite (positive semi definite, respectively).
- For a scalar function $V(x) > 0, (V(x) < 0)$ if $V(x)$ is positive definite (definite negative, respectively).
- $Qj, j = 1, \ldots, n$, denotes the $j$th row of matrix Q.
- $A \succeq 0$ stands for a *positive* matrix $A$, that is, a matrix with nonnegative elements: $a_{ij} \geq 0$.
- $\rho(A)$ stands for the radius spectrum of matrix $A$.
- $\mathscr{I}_r = \{1, \ldots, r\}$, while $\mathscr{I}_r^2 = \mathscr{I}_r \times \mathscr{I}_r$.
- A matrix whose its off-diagonal entries are non positive is called Z-matrix.
- $\mathbb{I}$ denotes the identity of appropriate size.
- For a square matrix $H \in \mathbb{R}^{m\times m}$, $\tilde{H}_d, \widetilde{H}_c \in \mathbb{R}^{2m\times 2m}$ are defined as

$$\tilde{H}_d = \begin{bmatrix} H^+ & H^- \\ H^- & H^+ \end{bmatrix},$$

$$\widetilde{H}_c = \begin{bmatrix} H_1 & H_2 \\ H_2 & H_1 \end{bmatrix}$$

with

$$H^+(i,j) = h_{ij}^+ = \text{Sup}(h_{ij}, 0), \; H^-(i,j) = h_{ij}^- = \text{Sup}(-h_{ij}, 0),$$

for $i, j = 1, \ldots, n$, where $h_{ij}$ denotes the matrix component $H(i,j)$ and

$$H_1(i,j) = \begin{cases} h_{ij} & \textit{if} \quad i = j \\ h_{ij}^+ & \textit{if} \quad i \neq j \end{cases} \quad H_2(i,j) = \begin{cases} 0 & \textit{if} \quad i = 0 \\ h_{ij}^- & \textit{if} \quad i \neq j \end{cases}$$

- $\mathbb{R}_+^n$ denotes the non-negative orthant of $n$-dimensional real space $\mathbb{R}^n$.
- $A^T$ denotes the transpose of real matrix $A$.
- A matrix $A \in \mathbb{R}^{n \times n}$ is called a *Metzler* matrix if its off-diagonal elements are nonnegative. That is, if $A = \{a_{ij}\}_{i,j=1}^n$, $A$ is Metzler if $a_{ij} \geq 0$ when $i \neq j$.
- A matrix $A$ (or a vector) is said to be nonnegative if all its components are nonnegative (by notation $A \geq 0$). It is said to be positive if all its components are positive ($A > 0$).
- $Sym(A) = A + A^T$.

# Introduction and Preview

Fuzzy control systems have been presented as an important tool to represent and implement human heuristic knowledge to control a system. This theory is based on a class of fuzzy models proposed by the authors in [1], which were designed to describe nonlinear systems as a collection of Linear Time Invariant (LTI) models blended together with nonlinear functions. These functions, called weighting functions, may depend on measurable premise variables (inputs, outputs of the system or external variables, as in the case of linear parameter variable (LPV) systems) [2]. The Takagi–Sugeno (T–S) fuzzy structures, also called quasi-LPV systems, offers an efficient representation of nonlinear behaviors while remaining relatively simple compared to general nonlinear models [18–26]. Additionally, another advantage of this system representation is that some results developed in the linear framework can be extended to T–S fuzzy models [2]. Using the T–S Fuzzy models has caused research on fuzzy controller design to gain great interest in the field of automatic control in recent years. Based on T–S fuzzy models, a number of most important issues in fuzzy control systems have been addressed in the last years [33–44]. These include stability analysis [3–7], incorporation of the performance index [8] and others such as $H_1$ [9] and $H_2$ [10] controls, robustness [11, 12], numerical implementations [13, 14] and their applications [15]. To design fuzzy controllers, a strategy used by automatic control specialists is based on the concept of Parallel Distributed Compensation (PDC) [27, 28]. The main idea of controller design is to derive each control rule so as to compensate each rule of the fuzzy system [17]. The stability of T–S fuzzy models and the design of T–S fuzzy control laws are, in most cases, investigated via the direct Lyapunov approach leading to a set of Linear Matrix Inequalities (LMIs) [30–32], which can be solved efficiently by using the existing optimization techniques [16]. To find a solution to stability and stabilization issues in T–S fuzzy systems, the standard approach is based on looking for a common quadratic function that satisfies sufficient conditions to guarantee stability in the Lyapunov sense [29]. Most of these conditions can be converted into LMI constraints, solvable through convex optimization techniques. The inherent flexibility of the LMI approach allows to obtain fuzzy controllers that guarantee both stability and performance of closed-loop systems [4, 5, 11].

On the other hand, a major problem which is still inherent to all dynamical systems is the presence of actuator saturation. The class of systems with saturations has enjoyed great interest during the last three decades. Even for linear systems, this problem has been an active area of research for many years. Two main approaches have been developed in the literature.

- The first one, the so-called positive invariance approach, is based on the design of controllers which work inside a region of linear behavior where saturations do not occur (see [45, 46] and the references therein). One can also refer to the work of [47] where the synthesis of the controller is presented as a technique of partial eigenstructure assignment. This resolution was also associated to the constrained regulator problem. This technique has already been applied to fuzzy systems by [33] and [37]. This approach is referred to as unsaturating controller.
- The second approach allows saturations to take effect while guaranteeing asymptotic stability (see [48] and the references therein). This approach, where the control may be saturated, leads to a bounded region of stability which is ellipsoidal and symmetric. This region can be easily obtained by the resolution of a set of LMIs. In this case, the approach is referred to as saturating controller. In [49], besides the saturated character of the control, additional constraints on the increment or rate are taken into account. The first works on saturated fuzzy systems without delay can be found in [50] and [51].

This book, considers the problem of the presence of saturation on the control as an additional problem to delays by proposing two kinds of controllers:

- Unsaturating controllers which stabilize the system without tolerating saturation to take effect. These controllers work in a linear region of behavior.
- Saturating controllers which stabilize the system while tolerating saturation to take effect.

Almost all the works on the subject use state feedback control or dynamic output feedback control. Static output feedback for fuzzy systems generally leads to a great number of LMIs to be satisfied [52, 53]. The cone complementary method of [54] has recently been used for fuzzy systems with a common output matrix in [55].

In this book, we propose to write the nonlinear system described by T–S models in an equivalent form extending the result of [25]. The objective is to obtain a reduced number of LMIs for the design of the static output controller. We propose an Output PDC (OPDC) controller which is useful when only the output of the system is available. Using the quadratic Lyapunov method, sufficient conditions of global asymptotic stability are obtained leading to the synthesis of the controller gains by means of a set of reduced number of LMIs.

The most commonly available methods use a quadratic common Lyapunov function. However, as was recently proven for hybrid systems [56] and fuzzy systems [31, 32, 55] the use of a piecewise quadratic Lyapunov function, introduced earlier for uncertain linear systems in [57], leads to better results in the sense that a common quadratic Lyapunov function may not exist while a multiple one may. In [58], the authors showed that even for a hybrid system composed of two stable linear invariant subsystems, a common quadratic Lyapunov function does not always exist. They also derive necessary conditions for the existence of such function for a set of two subsystems only. Hence, many works can be found in the literature using multiple Lyapunov functions with state or output feedback controls. These methods generally extend the obtained results with a common Lyapunov quadratic function, such as PDC (Parallel Distributed Compensation) control for continuous-time fuzzy systems [25] and non-PDC control for discrete-time fuzzy systems [41, 42, 59].

This book considers an additional problem usually found in dynamical systems: the nonnegativity of the states. The study of systems with nonnegative states is important in practice because many chemical, physical and biological processes involve quantities that have intrinsically constant and nonnegative sign: concentration of substances, level of liquids, etc, are always nonnegative. In the literature, systems whose states are nonnegative whenever the initial conditions are nonnegative are referred to as positive [60]. The design of controllers for these positive systems has been studied by [61, 62] where the authors provide a new treatment for the stabilization of positive linear systems. All the proposed conditions are necessary and sufficient, and expressed in terms of Linear Programming (LP). These results have then been extended to systems with delay by [63, 64]. One can think that LMI techniques can easily handle this new constraint of nonnegativity of the states. Nevertheless, this is not usually possible without taking onto account the use of the adequate Lyapunov function. The model of a real plant is used to show the need of such controllers in practice, especially for fuzzy systems where the model is global involving the hole state and not a state of variation around a set point. This idea, which was earlier used for positive switching systems in [65, 66], has a different impact on positive fuzzy systems due to the form of the obtained global matrix in closed-loop. Sufficient conditions of asymptotic stability for positive discrete-time fuzzy systems represented by T–S models were obtained for the first time in [67] with multiple Lyapunov functions. The idea developed by [61, 62] has been extended to T–S fuzzy systems in [68, 69] for continuous-time systems. However, for discrete-time T–S fuzzy systems, this book presents new results leading to design methods of stabilizing controller based on Linear programs instead of LMIs. Hence, it is shown that these methods are less conservative than LMI methods.

Recently, robust control and quadratic stabilization for linear systems with uncertain parameters have been considered in [70–73]. For fuzzy systems without uncertainties, in [74] Liu and Zhang have proposed a new design method based on the $H_\infty$ norm. However, their technique is based on the two-step approach which appears to be a drawback. An improvement of the control design method is proposed in [75]. In [76–78], robust observer-based control problems for uncertain fuzzy systems have been considered. However, the proposed design methods also use two steps to resolve the stability conditions. Like in [75], we propose a method to simplify and to improve the existing design methods of robust fuzzy state feedback stabilizing controllers based on fuzzy observer with disturbance attenuation for uncertain Takagi–Sugeno fuzzy systems. The developed method gives not only the controller and the observer gains on a single step but also a less conservative stability conditions.

On the other hand, time-delay often occurs in various practical control systems, such as transportation systems, communication systems, chemical processing systems, environmental systems and power systems [79]. The existence of delays may deteriorate the performances of the system and can be a source of instability [80]. As a consequence, the T–S fuzzy model has been extended to deal with nonlinear systems with time-delays [81–83]. The existing results of stability and stabilization criteria for this class of T–S fuzzy systems can be classified into two types: delay-independent conditions, which are applicable to delay of arbitrary size [84–89], and delay-dependent conditions, which include information on the size of delays, [85, 90–93]. It is generally recognized that delay-dependent results are usually less conservative than delay-independent ones, especially when the size of delay is small. We notice that all the results of analysis and synthesis delay-dependent methods cited previously are based on a single LKF that bring conservativeness in establishing the stability and stabilization tests. Moreover, the model transformation, the conservative inequalities and the so-called Moon's inequality [40] for bounding cross terms used in these methods also bring conservativeness. Recently, in order to reduce conservatism, the free weighting matrix technique has been proposed, originally by He et al. in [94, 95]. These works studied the stability of linear systems with time-varying delays. More recently, Huai-Ning et al. [16] have studied the problem of stabilization via PDC control by employing a fuzzy LKF combining the introduction of free weighting matrices which improve existing ones in [85, 93] without imposing any bounding techniques on some cross product terms. In general, the disadvantage of this new approach [16] lies in that the delay-dependent stabilization conditions presented involve three tuning parameters. Chen et al. in [85, 96] have proposed delay-dependent stabilization conditions of uncertain T–S fuzzy systems. The drawback in these works is that the time-delay must be constant. We note that the T–S fuzzy affine systems with delay is also studied in [97].

In this book, the asymptotic stabilization of uncertain T–S fuzzy systems with time-varying delay is studied. We focus on the delay-dependent stabilization synthesis based on the PDC scheme [98, 25]. Different from the methods currently found in the literature [16, 96, 99, 100], the proposed method does not need any transformation in the LKF, and thus, avoids the restriction resulting from any used transformation. This new approach improves the results in [16, 85, 93, 101] for three great main aspects. The first one concerns the reduction of conservatism. The second one, the reduction of the number of LMI conditions which reduce computational efforts. The third one, the delay-dependent stabilization conditions presented involve a single fixed parameter. This new approach also improves the work of Chen et al. in [96] by establishing new delay-dependent stabilization conditions of uncertain T–S fuzzy systems with time varying delay.

Almost all, the works on delayed T–S fuzzy systems considered a slowly varying time delay ($\dot{\tau}(t) < 1$). However, there are a number of practical time-varying delayed systems, such as traffic flow in communication networks and network controlled systems which belong to the class of fast time-varying delayed systems. In [102–105] a stabilization synthesis based on PDC control for both slowly and fast time varying-delayed systems has been designed. The problem of design of delay-independent observer-based $H_\infty$ Control for T–S fuzzy systems with time varying delay has been discussed in [106] and [107]. Although it is well known that delay-dependent results are less conservative than delay independent ones (particularly when the size of delay is small), there are few delay dependent results which study the problem of observer-based $H_\infty$ Control for T–S fuzzy systems with varying time delay. For example, Lin et al. in [108] proposed a delay-dependent work. But, the obtained results are limited to slow time varying delay systems. Furthermore, the problem is solved via utilizing a cone complementarity minimization algorithm which leads to significant computational demands. So far, to the best of our knowledge, there has been no delay-dependent method reported to study the observer-based $H_\infty$ control for T–S fuzzy systems with fast time varying delay. This motivates the research in this book to study this problem. The first advantage of the proposed result is that the $H_\infty$ controller design based on fuzzy observer conditions is formulated in terms of strict LMIs which can easily be solved in one step by using available software packages. The second one concerns the improvement of the restrictive results for delays with derivatives not greater than $1(\dot{\tau}(t) < 1)$ (fast time-varying delay fuzzy systems).

The method in this book uses a matrix decoupling technique as in [106, 107, 108]. Even for linear systems, observers have played an important rule in control theory over the last three decades [109–111]. Some works extend this problem to nonlinear systems [112–118]. During the last decade, many works interested to observers by using T–S fuzzy models have appeared in the literature (see [119] and the reference therein). Concerning the fuzzy observer based fuzzy control, [120] presents a two step approach which has been improved in [75] by using a matrix decoupling technique to establish strict LMI conditions based on a single step approach. For the uncertain case, a $H_\infty$ controller based on the fuzzy observer design method using a two step algorithm is proposed in [121]. This last result is improved in [23] by proposing the strict LMI conditions which are less conservative and can be resolved in one step.

The designing of observer-based fuzzy control and the introduction of performance guaranteed cost for T–S with input delay have been discussed in [122] and [123], respectively.

In [92, 124] and the references therein, stability analysis and synthesis based on the PDC scheme have been discussed. The observer based fuzzy control was treated in [105, 108] and the references therein. In [125], the problem of $H_\infty$ exponential stabilization was developed.

In last two decades, the two-dimensional (2D) system theory has been paid a considerable attention by many researchers. The 2D linear models have been introduced in the seventies [126, 127] and have found many applications, such as in digital data filtering, image processing [128], modeling of partial differential equations [129], etc. In connection with Roesser [128] and Fornasini–Marchesini [130] models, some important problems such as realization, controllability, minimum energy control, have been extensively investigated (see for example [131]). On the other hand, the stabilization problem has not been fully investigated and still not completely solved.

The stability of 2D discrete linear systems can be reduced to checking the stability of 2D characteristic polynomial [132, 133]. This appears to be a difficult task for the control synthesis problem. In the literature, various types of easily checkable but only sufficient conditions for asymptotic stability and stabilization problems for 2D discrete linear systems have been proposed [134–138].

This book is also interested to nonlinear 2D model Roesser systems described with 2D T–S fuzzy models. The obtained fuzzy system is then a set of $r$ linear 2D systems linked between them by membership functions. The objective of this work is the design of stabilizing controllers for this class of systems. To the best of our knowledge, no works have directly considered fuzzy 2D systems in the past except for the works of the authors. To this end, common Lyapunov quadratic and multiple Lyapunov functions are used. In this context, sufficient conditions of stabilizability are presented. Furthermore, these conditions are presented in the form of a set of LMIs for the state feedback control case. The first results on this topic have been presented in [139, 140].

This book is composed of nine chapters. The Chap. 1 presents the tools of T–S representation for nonlinear systems together with the basic results on stability and stabilization for continuous-time and discrete-time systems.

Chapter 2 deals with the extension of the positive invariance approach to nonlinear systems modeled by T–S fuzzy systems. The saturations on the control are taken into account during the design phase. Sufficient conditions of asymptotic stability are given ensuring in the same time that the control is always admissible inside the corresponding polyhedral set. Both a common Lyapunov function and piecewise Lyapunov function are used.

Chapter 3 presents a static output feedback controller design method for nonlinear systems represented by a T–S fuzzy model. Using the PDC structure, A new quadratic stabilization result is developed to design an output PDC (OPDC) controller. Based on the well known existing method in the literature, two methods are proposed. The design of the controller by static output feedback is given by two different sets of LMIs. Two examples are presented to illustrate these results.

Chapter 4 deals with sufficient conditions of asymptotic stability and stabilization for nonlinear discrete-time systems represented by T–S fuzzy models whose state variables take nothing but nonnegative values at all times for any nonnegative initial state. This class of systems is called positive T–S fuzzy systems. The conditions of stabilizability are obtained with state feedback control. This work is based on multiple Lyapunov functions. The results are presented in the LMI form. A real plant model is studied to illustrate this technique.

Chapter 5 deals with the problem of delay-dependent stability and stabilization of T–S fuzzy systems with a time-varying delay while imposing positivity in closed-loop. The stabilization conditions are derived using a single Lyapunov–Krasovskii Functional (LKF) combining the introduction of free-single matrices. A memory feedback control is also used in case the delay matrix is not nonnegative. An example of a real plant is studied to show the advantages of the design procedures.

Chapter 6 aims at designing a controller to robustly stabilize the uncertain nonlinear system with time-varying delay and norm bounded uncertainties via a T–S fuzzy model. The stabilization conditions are given in the form of LMIs using a single LKF combining the introduction of some relaxation matrices and only one tuning parameter. In comparison with the existing techniques in the literature, the proposed approach offers two major advantages. The first one is the reduction of computational complexity when the number of IF-THEN rules, r, is big. The second one concerns the conservatism reduction. Several examples are given to show the effectiveness and the merits of the design procedure.

Chapter 7 addresses the robust observer based $H_\infty$ control problem for T–S fuzzy systems with time-varying norm bounded uncertainties. Sufficient relaxed conditions for synthesis of a fuzzy observer and a fuzzy controller for T–S fuzzy systems are derived in terms of a set of LMIs. In comparison with the existing techniques in literature, the proposed approach considerably simplifies the design procedure and gives in only one step the controller and the observer gains. The observer and controller designed are capable to reject the disturbance assumed known but norm bounded. In order to highlight the performance of the proposed control algorithm, numerical simulations are performed.

Chapter 8 deals with the problem of stabilization by state feedback control of T–S fuzzy discrete-time systems with multiple fixed delays while imposing positivity in closed-loop. The obtained results are presented under LP form. In particular, the synthesis of state feedback controllers is first solved in terms of LP for the unbounded controls case. This result is then extended to the stabilization problem by nonnegative controls, and stabilization by bounded controls. The stabilization conditions are derived using the single LKF. An example of a real plant is studied to show the advantages of the design procedure. To show the merit of the proposed method, a comparison between LP and LMI approaches is presented upon a second example.

Chapter 9 deals with sufficient conditions of asymptotic stability for nonlinear discrete-time 2D systems represented by a Takagi–Sugeno fuzzy model of Roesser type with state feedback control. This work is based on common and multiple Lyapunov functions. The results are presented in LMI form. Continuous systems and discrete systems are both studied. 2D continuous fuzzy systems with delays have obtained a particular interest. Also, PDC control and non PDC control are both studied to show the limit of the PDC control while using multiple Lyapunov functions.

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