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Luca Salasnich

Quantum Physics of Light and Matter

A Modern Introduction to Photons,
Atoms and Many-Body Systems

 Springer

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Preface

This book contains lecture notes prepared for the one-semester course “Structure of Matter” belonging to the Master of Science in Physics at the University of Padova. The course gives an introduction to the field quantization (second quantization) of light and matter with applications to atomic physics.

Chapter 1 briefly reviews the origins of special relativity and quantum mechanics and the basic notions of quantum information theory and quantum statistical mechanics. **Chapter 2** is devoted to the second quantization of the electromagnetic field, while **Chap. 3** shows the consequences of the light field quantization in the description of electromagnetic transitions. In **Chap. 4**, it is analyzed the spin of the electron, and in particular its derivation from the Dirac equation, while **Chap. 5** investigates the effects of external electric and magnetic fields on the atomic spectra (Stark and Zeeman effects). **Chapter 6** describes the properties of systems composed by many interacting identical particles. It is also discussed the Fermi degeneracy and the Bose–Einstein condensation introducing the Hartree–Fock variational method, the density functional theory, and the Born–Oppenheimer approximation. Finally, in **Chap. 7**, it is explained the second quantization of the nonrelativistic matter field, i.e., the Schrödinger field, which gives a powerful tool for the investigation of finite-temperature many-body problems and also atomic quantum optics. Moreover, in this last chapter, fermionic Fock states and coherent states are presented and the Hamiltonians of Jaynes–Cummings and Bose–Hubbard are introduced and investigated. Three appendices on the Dirac delta function, the Fourier transform, and the Laplace transform complete the book.

It is important to stress that at the end of each chapter there are solved problems which help the students to put into practice the things they learned.

Padova, January 2014

Luca Salasnich

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