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Mathematical Models and Numerical Simulation in Electromagnetism

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To our students

Preface

This monograph has been developed as a basic support for a graduate course in electromagnetism oriented to numerical simulation. Thus, the emphasis is on the general equations rather than in solving analytically particular problems. The main goal is that the reader learns about the boundary-value problems of partial differential equations that should be solved, in general by using numerical methods, in order to perform computer simulation of electromagnetic processes. The book is more oriented to electrical engineering applications rather than to wave propagation problems related to telecommunication devices. Unlike most books on the subject, it goes from the general to the specific, namely, from the full Maxwell's equations to the particular cases of electrostatics, direct current, magnetostatics and eddy currents models. It also contains a first part devoted to electrical circuit theory based on ordinary differential equations. Apart from standard exercises related to analytical calculus, some others oriented to real-life applications are also included.

The book is structured in three parts. The first one is devoted to lumped parameter models so it includes an elementary circuit theory. It consists of three chapters. Chapter 1 is devoted to recall the harmonic oscillator. The use of complex functions to integrate the model is emphasized and the notion of impedance is introduced. In Chap. 2 the mechanical-electrical analogy is exploited to study series RLC circuits. Chapter 3 concerns more general linear circuits. They are modelled as digraphs leading to linear systems of integro-differential equations. Numerical solution is addressed both in the time and in the frequency domains.

The second part concerns distributed parameter models. In Chap. 4 Maxwell's equations in the empty space are recalled. Then they are reduced to vector wave equations in terms of either the electric field or the magnetic induction. The equations for the harmonic case are also obtained and then plane wave solutions are calculated. Chapter 5 includes several examples that can be solved by hand and Chap. 6 extends Maxwell's equations to material media. Concepts as electric polarization and magnetization are described and their influence in the electromagnetic field is analyzed. Next, the particular cases of electrostatics, direct current, magnetostatics and eddy currents are considered from Chaps. 7 to 10. The weak formulation of each problem most suitable to perform computer simulation of electromagnetic processes is chosen

and detailed. Moreover, Chap. 10 includes a brief section on coupling distributed and lumped models. The last chapter of Part II is devoted to nonlinear magnetics. This is the subject of Chap. 11 where we recall the hysteresis behavior and its mathematical modelling following the Preisach methodology.

The third part of the book is devoted to numerical simulation of electromagnetic problems. The numerical analysis of Maxwell's equations has been extensively developed during the last decades by mathematicians and engineers. Thus we can mention the books by Alonso and Valli [4], Bossavit [23], Chari and Silvester [31], Jin [46], Monk [55], Silvester and Ferrari [64], just to list a representative sampling of books dealing with this subject.

Numerical methods can address situations that otherwise would become totally unmanageable given the complexity or impracticability of its analytical solution. Moreover, with a suitable post processing of the results, it is possible to qualitatively and quantitatively understand what is really happening in the system under study. Among the numerical methods found in the literature to approximate Maxwell's equations, the finite element method (FEM) is the most extended. Its main advantages are its geometric flexibility and the richness in theoretical mathematical tools useful to analyze the approximation of the problem.

In recent years, the proliferation of commercial software packages attests to the growing use of numerical simulation in industry. This is due, in large part, to its usefulness in the design process of products reducing the "trial and error" cycle by evaluating multiple designs in the computer.

Among the commercial programs for the simulation of electromagnetic problems, we can mention Flux 2D and 3D from Cedrat [69], Ansys Multiphysics [68], Magnet from Infolytica [71], Opera from Cobham [73] or Comsol Multiphysics [70], to name the most used in low-frequency applications. Usually, due to their high cost, the use of these packages is mainly restricted to the research field. However, computer programs and packages are also increasingly being integrated into the graduate and undergraduate curriculum, serving as virtual laboratories to teach and reinforce concepts.

In this book we will use MaxFEM [72], a free software package developed in the Research Group of Mathematical Engineering from Universidade de Santiago de Compostela (USC), Spain.

MaxFEM is an open software for numerical simulation in electromagnetism based on the finite element method. It contains several applications covering electrostatics, direct current, magnetostatics and eddy currents in two and/or three dimensions and in Cartesian and/or cylindrical coordinates. In addition to the code itself, MaxFEM includes a user interface to facilitate its handling and it is fitted with an on-line help manual. It can be downloaded from <http://sourceforge.net/projects/maxfem/>.

By introducing these companion computational tools in the textbook, our goal is to strike a balance between the classical teaching of electromagnetism, which emphasizes its physical and mathematical foundations, and a numerical simulation approach which focuses on solving realistic problems using software packages.

With this goal in mind, and according to Parts I and II, we have structured Part III of the book in five chapters from 12 to 16 devoted, respectively, to electrical circuits, electrostatics, direct current, magnetostatics and eddy currents problems.

Finally, some basic and complementary subjects like graph theory, vector calculus or function spaces are included in appendices as well as Maxwell's equations in Lagrangian coordinates.

Most of the research in mathematical electromagnetism of the authors has been developed in collaboration with professor Rodolfo Rodríguez from the University of Concepción (Chile). Along many years we have enjoyed his intelligence, kindness and friendship. Some of our common former students, as Bibiana López and Pablo Venegas have carefully read several chapters of the manuscript or written computer programs now included in MaxFEM. We also express our gratitude to the rest of the team that has written this software: M. Carmen Muñoz, Francisco Pena, Marta Piñeiro, Víctor Sande and Rodrigo Valiña. Finally, we also acknowledge the excellent work by Marta Piñeiro and Víctor Sande for providing us with several examples and figures included in the book.

Santiago de Compostela and Lugo
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