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Constrained Control of Uncertain, Time-Varying, Discrete-Time Systems

An Interpolation-Based Approach



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To Linh and Tung

Preface

A fundamental problem in automatic control is the control of uncertain and/or time-varying plants with input and state or output constraints. Most elegantly, and theoretically most satisfying, the problem is solved by optimal control which, however, rarely gives a feedback solution, and oftentimes only a numerical solutions.

Therefore, in practice, the problem has seen many ad-hoc solutions, such as *override control*, *anti-windup*. Another solution, that has become popular during the last decades is *Model Predictive Control* (MPC) where an optimal control problem is solved at each sampling instant, and the element of the control vector meant for the nearest sampling interval is applied. In spite of the increased computational power of control computers, MPC is at present mainly suitable for low-order, nominally linear systems. The robust version of MPC is conservative and computationally complicated, while the *explicit* version of MPC that gives a piecewise affine state feedback solution involves a very complicated division of the state space into polyhedral cells.

In this book a novel and computationally cheap solution is presented for uncertain and/or time-varying linear discrete-time systems with polytopic bounded control and state (or output) vectors, with bounded disturbances. The approach is based on the interpolation between a stabilizing, outer low-gain controller that respects the control and state constraints, and an inner, high-gain controller, designed by any method that has its robustly positively invariant set satisfying the constraints. A simple Lyapunov function is used for the proof of closed loop stability.

In contrast to MPC, the new interpolating controller is not necessarily employing an optimization criterion inspired by performance. In its explicit form, the cell partitioning is considerable simpler than the MPC counterpart. For the implicit version, the on-line computational demand can be restricted to the solution of at most two linear programming problems or one quadratic programming problem or one semi-definite programming problem.

Several simulation examples are given, including uncertain linear systems with output feedback and disturbances. Some examples are compared with MPC. It is

believed that the new controller might see wide-spread use in industry, including the automotive industry, also for the control of fast, high-order systems with constraints.

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Notation¹

Sets

\mathbb{R}	Set of real number
\mathbb{R}_+	Set of nonnegative real number
\mathbb{R}^n	Set of real vectors with n elements
$\mathbb{R}^{n \times m}$	Set of real matrices with n rows and m columns

Algebraic Operators

A^T	Transpose of matrix A
A^{-1}	Inverse of matrix A
$A > (\geq) 0$	Positive (semi)definite matrix
$A < (\leq) 0$	Negative (semi)definite matrix

Set Operators

$P_1 \cap P_2$	Set intersection
$P_1 \oplus P_2$	Minkowski sum
$P_1 \ominus P_2$	Pontryagin difference
$P_1 \subseteq P_2$	P_1 is a subset of P_2
$P_1 \subset P_2$	P_1 is a strict subset of P_2
$P_1 \supseteq P_2$	P_1 is a superset of P_2
$P_1 \supset P_2$	P_1 is a strict superset of P_2
∂P	The boundary of P
$\text{Int}(P)$	The interior of P
$\text{Proj}_x(P)$	The orthogonal projection of the set P onto the x space

Others

I	Identity matrix of appropriate dimension
1	Matrix of ones of appropriate dimension
0	Matrix of zeros of appropriate dimension

¹The conventions and the notations used in the book are classical for the control literature. A short description is provided in the following.

Acronyms

LMI	Linear Matrix Inequality
LP	Linear Programming
QP	Quadratic Programming
LQR	Linear Quadratic Regulator
LTI	Linear Time Invariant
LPV	Linear Parameter Varying
PWA	PieceWise Affine