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Saeed Eftekhar Azam

# Online Damage Detection in Structural Systems

Applications of Proper Orthogonal  
Decomposition, and Kalman  
and Particle Filters



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# Foreword

Monitoring the health of structures and infrastructures exposed to aging or extreme loadings is nowadays recognized as a societal need. The pervasive use of miniaturized sensors, recently developed through microelectronics-driven technological processes, has forced people to look for smart monitoring strategies tailored to handle the large amount of data provided by densely deployed sensor networks. Moreover, as each health monitoring procedure relies upon a theoretical/numerical model of the considered structure, the more accurate the model the more powerful the monitoring scheme; such increased accuracy also entails additional monitoring burden.

If a structure undergoes a damaging process reducing its load-carrying capacity, the health monitoring procedure should be able to identify the damage itself in terms of location and amplitude. It is then necessary to filter out the possible noise terms and provide meaningful information from the structural response. Due to the presence of damage, robust procedures able to deal with a nonlinear system evolution are obviously to be envisioned.

The two topics discussed above, i.e., the size of the model to be handled and the nonlinearities in its evolution law, might be difficult to manage simultaneously in a common frame. It may happen that the filtering algorithm, which is supposed to compare the responses of the real structure and of a fictitious, linear-comparison one (featuring no damage evolution in a predefined time window), provides estimations affected by drifts or biases, sometimes also diverging. It may also happen that by increasing the size of the numerical model, e.g., due to a required finer space discretization in case of numerical (e.g., finite element) procedures supplying the model itself, the aforementioned bias and divergence issues get amplified.

It is also worth noting that structural health monitoring systems should be able to provide results in real-time or, at least, close to such target, so that warnings can be provided as soon as critical conditions are approached during the life cycle of a structure.

The research activity reported in this book moved from all the aforementioned critical aspects, with the aim of providing a robust, accurate, and easy to implement methodology for the health monitoring of civil structures and infrastructures, possibly suffering damage inception and growth. Two main challenging topics are

specifically dealt with: the derivative-free filtering of the response of nonlinear systems; a time-varying, reduced-order modeling able to self-adapt to a changing system dynamics. As for the former issue, results are known to be not satisfactory if one does not properly account for the statistics of noise terms and structural state, and for the nonlinear evolution of the last ones. Here, the author shows that a wise combination of Kalman and particle filtering can indeed provide a very efficient (in terms of computational costs) and robust (in terms of avoidance of output divergence) framework. As for the latter issue, a snapshot-driven proper orthogonal decomposition methodology is known to work well in case of time-evolving linear systems; on the other hand, it is still disputed whether proper orthogonal decomposition can be adopted for a nonlinear time evolution of the system, linked, e.g., to damage growth. Here, the author shows that a further exploitation of Kalman filtering can provide, if governed by a partial observation of the system, a very efficient way to continuously tune the reduced-order model, thereby avoiding time-consuming re-training stages suggested by others in the past.

This book thus introduces a novel, hybrid approach to damage identification and health monitoring of structural systems. As such, it has been written mainly focusing on the theoretical and implementational aspects of the approach, partially leaving experimental validations aside. In my opinion, readers can find in it all the details necessary to adapt the methodology to many, if not all the real-life situations to be practically envisioned.

Stefano Mariani

# Preface

The aim of this monograph is to present the key ingredients of a still-in-progress research discipline within the structural engineering realm, namely online damage detection. The material of the text offers detailed explanations on recursive Bayesian filters (e.g., Kalman filters, particle filters), proper orthogonal decomposition methods (POD) (e.g., singular value decomposition, principal component analysis), and a combination thereof, i.e., a synergy of reduced order modeling and recursive Bayesian filtering. Illustrations accompanied by the theoretical description allow the reader to intuitively comprehend the notions. Therefore, this book can serve as a tutorial for scientists and engineers who want to apply and implement proper orthogonal decomposition and/or Bayesian filters to a specific problem.

Throughout the book, the focus of the numerical examples is on structural systems. The techniques presented in this research monograph are well established in fields like automatic control, statistics, etc. However, they are rather new to civil and structural engineers; hence, the algorithms are presented in enough details so that the reader can easily implement them on any structural state-space model. At first, the ease of implementation has been the main concern; however, the author believes that the way the main notions are analyzed makes this book an inspiration for conducting further research and development of these methods.

The objective of the study presented in this monograph is to develop techniques for vibration-based non-destructive damage identification of the structures. In fact, the major emphasis is on the development of quick and robust recursive damage detection algorithms in order to facilitate the task of online, real-time continuous monitoring of civil structures, such as, e.g., residential buildings, bridges, and other similar structures. This goal can be accomplished only through mixing different disciplines of science and technology, including automatic control, applied mathematics, and structural engineering.

It should be emphasized that though Bayesian filters have been extensively studied in the automatic control field, their applications in structural engineering are yet to be investigated. The applications of extended Kalman filter (EKF), sigma-point Kalman filter (SPKF), and particle filter (PF) to simplified and low-dimensional models are suggested in the existing literature; nevertheless, to the best of my knowledge, applying the extended Kalman-particle filter (EK-PF) has never been reported when dealing with a structural engineering problem.

The algorithms for all the Bayesian filters used in this book are derived using the same notation; this can allow the reader to easily understand the similarities and ideas behind each one of them. Their performances dealing with different identification tasks are scrutinized in detail, and the reason for their success and failure in each case is highlighted.

It is perceived that as the number of the degrees-of-freedom increases, the adopted methods in the literature lose their accuracy in system identification, and thus in damage detection process. This problem is created due to the high dimension of the parameter space, i.e., by so-called curse of dimensionality. To manage this issue, in this study I make recourse to reduced order modeling of the systems. The aforementioned task is accomplished by using the proper orthogonal decomposition. Before using POD-based models in the Bayesian filters, the performance of such methodology is thoroughly investigated to ensure accuracy, speed-up, and robustness when different sources of excitation shake the structures.

The major contribution of the present research is the development of a recursive stochastic algorithm by a synergy of dual estimation concept, POD-based order reduction, and a subspace update. The proposed methodology takes advantage of Bayesian filters (like EKF and EK-PF) for dual estimation of state and parameters of a reduced order model of a time-varying system. A Kalman filter is employed within each iteration period to update the subspace spanned by the POMs of the structure. The efficiency and effectiveness of the algorithm are verified via pseudo-experimental tests conducted on multi-storey shear buildings. It will be shown that the procedure successfully identifies the state, the model parameters (i.e., the components of the reduced stiffness matrix of the structure) and relevant proper orthogonal modes (POMs) of the reduced model. Unbiased estimates furnished by the algorithm permit the health monitoring of the structure.

By reading this monograph, one could learn how the family of Kalman filters and particle filters are connected; compare their performances when dealing with a structural dynamics problem; see through detailed examples why and when they fail; figure out which filter can better fit a certain problem; and know how to tune the parameters of the filters. Moreover, the way the filters are presented renders the task of implementing more complicated filters easy and even developing ad hoc filters for structural engineering possible. Concerning reduced order modeling, possible limitations caused by POD-based reduced models are shown via numerous graphs and tables. The nature and extent of the inaccuracies caused by abridging the full mathematical model of the structures are carefully studied and analyzed. Finally, the use of such reduced models in the Bayesian filters is studied for the case in which the model can change (sustain damage) and also when it is a priori known that the model remains undamaged.

To follow the contents of this monograph, the reader is expected to have a background in statistics and calculus, and to be familiar with linear algebra and fundamentals of signal processing.

The material covered in this book is derived from the doctoral dissertation of the author, which was submitted to the scientific faculty of the doctoral program at the Department of Structural Engineering of the Politecnico di Milano. The author

sincerely acknowledges the role of his supervisor Prof. Stefano Mariani in shaping this monograph; his encouragements, friendliness, rational way of thoughts, mindful directions, and scientific attitude had been the main elements without which this work would never have been in its present state. The author wants to thank all his friends in the department and outside it, whose presence made those years so memorable.

Saeed Eftekhar Azam



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