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Gerber–Shiu Risk Theory

 Springer

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Preface

These notes were developed whilst giving a graduate lecture course (*Nachdiplomvorlesung*) in the fall of 2012 at the Forschungsinstitut für Mathematik (FIM), ETH Zürich, Switzerland. The same course was given in the spring of 2013 at Centro de Investigación en Matemáticas (CIMAT), Guanajuato, Mexico and, simultaneously by video-link, at Instituto de Matemáticas, UNAM in Mexico City. The title of these lecture notes may come as surprise to some readers as, to date, the term *Gerber–Shiu Risk Theory* is not widely used. One might be more tempted to simply use the title *Ruin theory for Cramér–Lundberg models* instead. However, my objective here is to focus on the recent interaction between a large body of research literature, spearheaded by Hans Gerber and Elias Shiu, concerning ever more sophisticated questions around the event of ruin for the classical Cramér–Lundberg surplus process, and the parallel evolution of the fluctuation theory of Lévy processes. The fusion of these two fields has provided economies to proofs of older results, as well as pushing classical theory much further into what one might describe as *exotic ruin theory*. The latter may be considered as the study of ruinous scenarios which involve perturbations to the surplus coming from dividend or tax payments that have a historical path dependence. These notes keep to the Cramér–Lundberg setting. However, the text has been written in a form that appeals to straightforward and accessible proofs, which take advantage, as much as possible, of the fact that Cramér–Lundberg processes have stationary and independent increments and no upward jumps.

I would like to thank Paul Embrechts for the invitation to spend six months at the FIM and the opportunity to develop and present this material. Similarly I would like to thank Maria-Emilia Caballero, Juan Carlos Pardo and Victor Rivero for the invitation to give the same course at CIMAT and UNAM simultaneously. I would also like to thank all attendees in Zürich, Guanajuato and Mexico City for their comments (especially Jean Bertoin, Leif Döring, Juan Carlos Pardo and Victor Rivero). Whilst I was in Switzerland I had the opportunity to meet Hans Gerber and Hansjörg Albrecher, with whom I had extensive discussion regarding some of the material in this book. I would also like to express my gratitude for their many observations and comments. I would also like to thank Nick Bingham and Erik Baurdoux who have

diligently attended to mathematical errors, references and my use of the English language. Finally, Springer produced three anonymous referee reports whose useful comments I am also grateful for.

Guanajuato, Mexico
May, 2013

Andreas E. Kyprianou

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