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Dario Villamaina

# Transport Properties in Non-Equilibrium and Anomalous Systems

Doctoral Thesis accepted by  
the Università La Sapienza, Italy

 Springer

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*To Isabella, who taught me what love is*

# Supervisors' Foreword

The study of fluctuations in statistical physics has a long history, starting from Einstein's theory, who was able to connect the fluctuations of macroscopic observables to the entropy. Later, in his seminal paper, Lars Onsager pointed out in a general context the deep relation between the relaxation of an external perturbation and the equilibrium fluctuations. A first example of a fluctuation/dissipation theorem had been derived by Einstein in his seminal work on Brownian motion, where the celebrated relation between mobility and diffusion was shown to be valid. The fluctuation/dissipation theorem and the linear response theory are fundamental aspects of statistical mechanics and have brought many important applications.

In the last decades, the development of modern technology has made possible the study of systems at the nanoscale, where fluctuations play a relevant role, ranging from the physics of colloids and small systems to biological inspired experiments, like single molecule manipulations and molecular motors. Quite surprisingly, when nonconservative forces or non-thermal fluctuations are present, a "simple" approach based on the standard equilibrium statistical mechanics fails to provide correct predictions for the observables of interest. Up to now a general theory of non-equilibrium fluctuations is still an open and challenging problem, being the main point that inspired the research presented in this book.

The work of Dr. D. Villamaina has the ambition to contribute toward the construction of such a general theory, by adding new interesting pieces to the puzzle. His thesis leads the reader from the analysis of simple models of heat conduction to the more realistic cases of granular materials and systems exhibiting anomalous transport. In all these cases the limits of the standard equilibrium approaches are discussed and the proper non-equilibrium fluctuation theory is derived. From the analysis of the different discussed systems, a general lesson does emerge: the correlations among different degrees of freedom can work as a channel of energy exchange, being responsible for the breaking of the equilibrium fluctuation theory. By following this line of reasoning, the author sheds light on the main source of irreversibility in the granular gases, being the cross correlations among the velocities of different particles. This theoretical prediction has also been experimentally verified in a series of recent publications.

In summary, the work of Dr. D. Villamaina shows in a convincing way how the study of non-equilibrium fluctuations and their relation with the response to an external perturbation are crucial for the construction of a proper and effective general theory. The research presented here can be extended to different classes of non-equilibrium and anomalous systems. In conclusion we are sure that this thesis will stimulate the debate, attracting new students and increasing the interest in this research field where many questions are still to be answered.

Rome, June 2013

Prof. Angelo Vulpiani  
Dr. Andrea Puglisi

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