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Inequalities for the Numerical Radius of Linear Operators in Hilbert Spaces

Springer

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To my granddaughter Audrey Elise

Preface

As pointed out by Gustafson and Rao in their seminal book [*Numerical Range. The Field of Values of Linear Operators and Matrices*. Universitext. Springer-Verlag, New York, 1997. xiv+189 pp.], the concepts of *numerical range* and *numerical radius* play an important role in various fields of contemporary mathematics, including operator theory, operator trigonometry, numerical analysis, and fluid dynamics.

Since 1997 the research devoted to these mathematical objects has grown greatly. A simple search in the database *MathSciNet* of the *American Mathematical Society* with the keyword "numerical range" in the title reveals more than 300 papers published after 1997 while the same search with the keyword "numerical radius" adds other 100, showing an immense interest on the subject by numerous researchers working in different fields of modern mathematics. If no restrictions for the year are imposed, the number of papers with those keywords in the title exceeds 1,000. However, the size of the areas of applications for numerical ranges and radii is very difficult to estimate. If we perform a search looking for the publications where in a way or another the concept of "numerical range" is used, we can get more than 1,550 items.

The present monograph is focused on numerical radius inequalities for bounded linear operators on complex Hilbert spaces for the case of one and two operators.

The book is intended for use both by researchers in various fields of linear operator theory in Hilbert spaces and mathematical inequalities, domains which have grown exponentially in the last decade, and by postgraduate students and scientists applying inequalities in their specific areas.

In the introductory chapter we present some fundamental facts about the numerical range and the numerical radius of bounded linear operators in Hilbert spaces. Some classical inequalities due to Berger, Holbrook, Fong and Holbrook and Bouldin are given. More recent and interesting results obtained by Kittaneh, El-Haddad and Kittanek and Yamazaki are provided as well.

In Chap. 2, we present recent results obtained by the author concerning numerical radius and norm inequalities for one operator on a complex Hilbert space. The techniques employed to prove the results are elementary. We also use some special vector inequalities in inner product spaces due to Buzano, Goldstein, Ryff and Clarke as well as some reverse Schwarz inequalities and Grüss type inequalities obtained by the author. Numerous references for the Kantorovich inequality that is extended to larger classes of operators than positive operators are provided as well.

In Chap. 3, we present recent results obtained by the author concerning the norms and the numerical radii of two bounded linear operators. The techniques in this case are also elementary and can be understood by undergraduate students taking a subject in operator theory. Some vector inequalities in inner product spaces as well as inequalities for means of nonnegative real numbers are also employed.

For the sake of completeness, all the results presented are completely proved and the original references where they have been firstly obtained are mentioned. The chapters are followed by the list of references used therein and therefore are relatively independent and can be read separately.

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