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Guglielmo Paoletti

Deterministic Abelian Sandpile Models and Patterns

Doctoral Thesis accepted by
the University of Pisa, Italy

Springer

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ISSN 2190-5053 ISSN 2190-5061 (electronic)
ISBN 978-3-319-01203-2 ISBN 978-3-319-01204-9 (eBook)
DOI 10.1007/978-3-319-01204-9
Springer Cham Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013942988

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

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Supervisor's Foreword

The Abelian Sandpile Model is a particular cellular automaton: discrete state space and dynamical evolution on discrete time. A collection of grains of sand is distributed among the vertices of a graph. A vertex is unstable if it has at least as many grains as its degree. An unstable vertex topples by giving one grain to each neighbor vertex. So that also some neighbor vertex which was stable can become unstable. But the stable configuration which is reached is independent from the particular order of the toppings. The configuration obtained after relaxation is the same if you add some grain of sand first here and afterward there or in the reversed order. For this reason the Sandpile is called Abelian.

It was originally introduced on a finite portion of the square lattice in such a way that sand is added randomly and eventually deposited outside such a region, so that a stable configuration is always reached. By looking at the statistical properties of the recurrent configurations reached in this way, the model appears as the simplest case of self-organized-criticality, that is a dynamical system which shows algebraic long-range correlations, without any tuning of parameters.

The model has been soon generalized to a generic graph. And a remarkable correspondence between recurrent configurations and trees on the graph has been discovered.

Very complex and amazingly beautiful periodic patterns are often generated by the dynamics, in particular in deterministic protocols in which the sand is added in chosen sites.

In his work Paoletti derived exact results which are potentially interesting also outside the area of critical phenomena. Exact means also site by site and not on ensemble averages or coarse graining.

For example, Paoletti studies the appearance of allometric structures, that is patterns which grow in the same way in their whole body, and not only near their boundaries, as commonly occurs.

On the plane, beyond patches, that is patterns periodic in both dimensions, one dimensional periodic structures, called strings, show up. They are classified by their principal periodic vector $\boldsymbol{k}$, that we call momentum. A simple relation between the momentum of a string and its density of particles, E, is obtained. This is reminiscent of a dispersion relation, $E = k^2$. Strings interact: they can merge and

split and within these processes momentum is conserved. The modular group $SL(2, \mathbb{Z})$ plays an important role behind these laws. And this reveals interesting connections with number theory.

Indeed, this kind of analysis has already become interesting not only in out-of-equilibrium statistical mechanics, but also in mathematics, both in probability as in combinatorics.

New questions arise in an old subject. Therefore I believe that on one side this thesis can amuse computer practitioners for the simplicity in which charming patterns can be obtained, on the other it can attract the interest of researchers working in many different areas.

I wish to add a personal note. It has been a pleasure to have a student like Guglielmo and to work in collaboration with Andrea Sportiello on this beautiful subject.

Milan, April 2013 Prof. Sergio Caracciolo

Acknowledgments

Here we are, at the end of my Ph.D. Thesis but also at the end of four years of my life here in Pisa, the end of an "*era*". It is funny how, simply changing the perspective, this is just the beginning of a new, big, adventure.

Acknowledgments are the only part of a Thesis every friend looks for when he open it. So, my friends, I am writing them for you and I hope you will like them, as much as I loved writing them.

First I want to thank my supervisor, Prof. Sergio Caracciolo, and my collaborator, Andrea S., who followed me wisely during these last few years. Their advises were fundamental in my research, as well as every discussion and dialogue we had made me grow. I feel very lucky to have had the possibility to work with them.

Then my thoughts go to my family, Raffaele, Adriana, and Rebecca, they all were so comprehensive to me, and I am sure that has been a tough task sometimes. They listened to my complaints, they spurred me on to greater efforts, and without the consciousness of their presence and their confidence in my skills I doubt I would have come to this point.

A special thank comes to Chiara and she knows why, cause any word would not be enough, a huge kiss (naic to you) and "Did I say that I need you"

How not say a word to Filippo who endured my presence and my good character in our three and more years as flatmates, and our friends and guest in via dell'Aeroporto 45: Emilio, Manu, Ale, Alberto, Sofia, Eleonora, Giangi, Davide, and Annalisa. Diners in via dell'Aeroporto are not finished, and I will join you once more.

Then Andrea, he is my Ph.D. mate. After our Ghezzano experience during these years we had the possibility to know each other, to travel around with the other Ghezzano boys, to party together with the German's guest and, last but not the least, to go lunch at "Circolino" (here is another place that I cannot forget and that I will bring in my memories wherever I will go, because "It is tough"). The lunches with Andrea, Katia, prof. Tonelli, Raffa, Elena will be object of many tales in the future. Thanks to you all and to "Circolino" I hope I will have the possibility to go there again whenever I will come back to Pisa.

Now, speaking about Ghezzano boys, a huge hug to Matteo, Damiano, and Claire, we all bring Ghezzano with us because it is not a place but a state of mind.

And also thanks for the good times together to Francesca, Umbe, Micche, Serena, Ciccio, Jacopo, Fabiolino, Laura, Lorenzo, Elisa, Ela, and Filippo.

During these last years in Pisa I must thanks the Caving group *GSPi*, cause there I made many friends, through them I discovered the Alpi Apuane and I also started to climb a bit. So thanks to Pascal, Giovanni (I will never forget our half marathon experience), Paolo, Andrea, Evelin, Teresa, il Pleb, Martina, Lara, Sandra, Il Giuntoli, Giorgio (il) Grande, Marco, Federico, Elena, Simone, and il Mancini.

Here in Pisa I also had the possibility to share my passion for photography with a group of amazing guys, with whom we founded a photo club "vividicontrasti", thanks to Cice (our artist), Fabio R., Fabio A., Nello, Gianni, Emmerre, lo Sfrek, Marta, Andrea, Chiara, Il Bini, Sara, Annalisa, and Dario.

I cannot forget to cite "Numeroundici" and Marco, the owner; in this little fantastic restaurant we had the first exhibitions of our photo club, but Marco is not only the guy behind the counter, he is a *friend* and I will always remember our chats and the photo sessions for the pictures of its website.

Although we all were far away, the old Milano aula *G* group is always in my mind, we met for holidays, parties around Europe, good times together in Milano, and I know we will meet soon, Marco, Luca Roma, Gio Viola, Jo, Ricky, Il Sanghi, Luca T., Elisa, and the Zanini brothers.

The same holds for all the guys from Varallo, you are always with me, wherever I go, I have been so happy every time I came back home to spend great time with you and I loved when you were guest at my place, I hope you will come also in my next destination! Thanks Paolo, Giovanni, Efrem, Cacaito, Elisa, Lucia, Nadia, Gigi, Laura, Andrea, Anna, Carla, Luz, Max, Agne, Jula, and Chicco.

I want also to thank all the physicists I met here in the Department and around the world in schools and congresses, we will meet again for sure, Bjarke, Walter, Matteo, Dario, Ezequiel, Benedetta, Carlos, Elia, Philippe, Roberto, Leone, Maurizio, Claudio, Vincenzo, Elio, and many others.

I already know I have forgot someone, someone who deserves a place in these Acknowledgments, please forgive me, I have a hug for all of you.

Guglielmo Paoletti

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