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Mathew Richard Bullimore

Scattering Amplitudes and Wilson Loops in Twistor Space

Doctoral Thesis accepted
by the University of Oxford, UK

Springer

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Supervisor's Foreword

It is a pleasure to write this foreword for Mat Bullimore's thesis. This research took place at an exciting time for the development of the subject and he was able to not only benefit from a collaboration with people in Oxford, but perhaps more importantly with David Skinner who had recently left Oxford for the Perimeter Institute and who played the role of an additional supervisor. There was also much fruitful interaction with the group led by Nima Arkani-Hamed and Freddy Cachazo in the IAS Princeton and Perimeter respectively. Of course this rapid development meant that, although there were opportunites, there was much for a new student to learn and it was necessary to be very quick to keep up with the competition.

Scattering amplitudes have always been a key output of quantum field theories. They are required in order to understand the physical consequences of particle models and to enable their predictions to be compared to experiment. Their study has become particularly pressing with the wealth of data arriving from the Large Hadron Collider for testing the standard model. However, the last 10 years has seen a completely different emphasis in the study of scattering amplitudes, arising from the unexpected mathematical structures that have been discovered that cry out for new physical and mathematical insights into the origins of physical theories.

Hints of such structures were first observed some time ago when some of the most elementary amplitudes, the so-called MHV amplitudes, turned out to have remarkably simple formulae despite the complexity of their origin as a sum of Feynman diagrams. More recently Witten showed that tree-level Yang-Mills amplitudes have a remarkably simple structure in twistor space and indeed could be understood as arising from a string theory in twistor space (as opposed to the more usual one in space-time). Twistor space is an auxilliary space that encapsulates the degrees of freedom of spinning massless particles in a complex three-dimensional projective space. This original twistor string was not viable as a model for conventional physics owing to the presence of conformal gravity. Nevertheless, the observation remained that, in twistor space, formulae for amplitudes are somehow simpler and more string like, being supported on curves of various degrees in twistor space.

There were a number of important outputs of these ideas. They included the MHV formalism, a particularly efficient Feynman diagram-like formalism for gauge theory amplitudes. Indeed, it had already emerged that the MHV formalism is indeed the Feynman diagram formalism for a gauge theory action principle in twistor space that has been expressed in an axial gauge. Another important set of tools are BCFW recursion relations (due to Britto, Cachazo, Feng and Witten) that determine more complicated amplitudes in terms of easier ones.

A major conjecture that had come to the fore in the run up to Mat's thesis was the surprising amplitude/Wilson duality: that planar amplitudes for N = 4 super Yang-Mills could computed as the correlation function of a Wilson loop. This had emerged from a string T-duality applied in the context of AdS/CFT that had initially seemed far fetched, but which worked out remarkably well in examples when details were worked out. Planar N = 4 super Yang-Mills became the optimal playground for trying to understand the mathematical structure of amplitudes.

The big open questions in these twistor related approaches was as to how one might study loop amplitudes and more general correlation functions using twistor theory. A halfway house between tree and loop ampltiudes are the so-called leading singularlities of loop amplitudes. A remarkable twistor-related generating formula for these had been obtained by the Princeton-Perimeter Institute collaboration in terms of a contour integral in a Grassmannian. His first paper gave a detailed analysis of the string-like structure of leading singularities in twistor space and the closely related expressions arising from BCFW recursion in twistor space, and the Grassmannian formulae was shown to arise partially as a consequence of such a twistor-string representation. This led to a classification scheme for leading singularities.

The next year saw further major developments in which it was seen how to gain a firmer grip on loop amplitudes via its integrand. On the one hand the Princeton/Perimeter collaboration were able to generate the all-loop integrand via an extension of BCFW recursion, whereas on the other, Mat with collaborators in Oxford and Perimeter was able to generate it using the MHV formalism in twistor space. This led to the realization that the amplitude/Wilson-loop duality could be expressed and proved perturbatively at the level of the integrand in twistor space from the twistor action. The Wilson loop was seen to correspond to holomorphic link invariants in twistor space and Mat, with David Skinner was able to show how the amplitude/Wilson-loop correspondence could be seen to follow from the loop equations adapted to holomorphic linking, a complex analytic generalization of real link invariants in three space.

The next year saw further important developments. One, not covered in the thesis but that Mat was involved with, was the observation that the twistor action framework was able to compute essentially arbitrary observables (although some with greater ease than others), and this allowed the proof of a correspondence between certain limits of certain correlation functions and null polygonal Wilson-loops, and hence planar amplitudes. The other was the study of symmetries and anomalies of the Wilson-loop. This led to certain descent equations that in principle allow the construction of integrated amplitudes. This was a great breakthrough as previously the methods had been restricted to computing integrands for correlation functions and amplitudes at loop level, but the anomalies led to descent equations that in principle at least, determines the integrated amplitude directly, and that could be used to compute highly nontrivial examples.

The subject is still evolving rapidly and promises many further new insights, but this thesis will remain a valuable resource, providing a detailed introduction and recording many of the key insights of the last few years.

Oxford, September 2013 Prof. Lionel J. Mason

Abstract

Scattering amplitudes are fundamental and remarkably rich observables in quantum field theory. The basic observation that makes scattering amplitudes fascinating is that in theories with massless particles of spin $s \geq 1$, they are much simpler than might be expected from traditional Feynman diagram techniques for computing them. This simple observation might ultimately have profound consequences for our view of quantum field theory. The broad aim of this thesis is to understand and exploit the hidden simplicity and structure in scattering amplitudes.

The quantum field theory with the simplest scattering amplitudes in four dimensions is planar $\mathcal{N} = 4$ supersymmetric Yang–Mills theory. This theory has provided considerable inspiration in developing new computational techniques and has provided many important theoretical insights. In this theory, there is a remarkable correspondence between scattering amplitudes and null polygonal Wilson loops, observables which on first inspection look very different. In this thesis, we will provide new insights into this correspondence using methods from twistor theory and exploit the symmetries of the problem to find new ways of computing scattering amplitudes.

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