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Complete Second Order Linear Differential Equations in Hilbert Spaces

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To the memory
of my grandparents
Max Rabur and
Dora Vainerman

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First order linear differential equations in Banach spaces

$$y'(t) + Ay(t) = 0 \tag{0.1}$$

and «incomplete» second order equations

$$y''(t) + By(t) = 0 \tag{0.2}$$

and related nonhomogeneous and nonlinear equations, as well as semigroups and cosine operator functions closely related to these equations, have been studied extensively during the second half of our century and have become to date a classical part of modern mathematical analysis (see, for example, the monographs [3, 57, 60, 92, 93, 131, 137, 150, 179, 211, 243, 268, 334, 375, 382] and the reviews [138, 180, 231, 352] and references therein).

The present monograph is an attempt to give a unified systematic theory of complete second order linear differential equations

$$y''(t) + Ay'(t) + By(t) = 0 \tag{0.3}$$

in Hilbert spaces, considering from a single point of view the following subjects: well-posedness of the Cauchy problem as well as the Dirichlet and Neumann problems; initial (boundary) conditions ensuring the solvability of initial (boundary) value problems; boundary behaviour and extension of solutions on a finite interval; stability and stabilization of solutions at infinity; boundary-value problems on a half-line. The theory is developed in a special but important case, which can be considered as model. We study equation (0.3) as an independent object without any references to equations (0.1) or (0.2).

Answers to all posed questions are given in the form of necessary and sufficient conditions. Particular emphasis is placed on new effects which do not take place for first order and incomplete second order equations.

To this end, we obtain some unexpected results relating to the spectral theory of pairs of operators and to the boundary behaviour of integral transforms (the latter results are considered as analogues of Tauberian theorems).

I hope that this monograph can also serve as an introductory course and a reference book in this subject. The presentation in the monograph is self-contained. The only prerequisite for its understanding is the knowledge of basic concepts and theorems on measure and integration, linear spaces, and linear operators in Banach and Hilbert spaces, according to the first course in functional analysis and operator theory. In particular, it is expected that the reader is familiar with the spectral theory for self-adjoint and, in general, normal operators in Hilbert space (recall that these operators do not need to be bounded or to have discrete spectrum). All necessary information may be found, for instance, in [287]; for an extended presentation see, for example, [2, 34, 75, 126, 141, 245, 355]. One of the new textbooks introducing all concepts, notations and results that are necessary for understanding this monograph is «Functional Analysis» (Vols. 1, 2) by Yu.M. Berezanskii, G.F. Us and Z.E. Sheftel just published in Birkhäuser's Operator Theory series.

A brief remark on notation. We will frequently use logical quantors; recall their meaning: «$\exists \ldots$» means «there exist(s)... such that»; «$\forall \ldots$» means «for an arbitrary (for all)...»; finally, «:» (used without a word before it) means «there holds the following condition:» («we have:»). The notation $\{x \in X | P(x)\}$ stands for the set of all those elements of X which satisfy the condition(s) $P(x)$.

I would like to emphasize a great impact on my investigations and the contents of the monograph made by Yu. M. Berezanskii and M. L. Gorbachuk.

My appreciation also to participants of seminars in the Institute of Mathematics of the National Academy of Sciences of Ukraine, especially to S. D. Eidelman, Yu. S. Samoilenko and G. F. Us, for useful discussions.

It is of great importance for me that S. G. Krein and J. A. Goldstein, authors of classical monographs on linear differential equations in Banach space, supported and encouraged this work.

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Kiev, February 1996 A. Ya. Shklyar