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Preface

This book consists of lecture notes for a nachdiplom course on disordered systems given at the mathematics department of ETH-Zürich in 1996. There were eleven lectures, each consisting of two 45 minute parts. The first lecture took place on April 18 and the last on July 5. Mario Wütrich assisted in the preparation of these notes with excellent editing and word processing, for which I express my gratitude.

In the physics literature, the term disordered system refers to a statistical mechanics model in a random environment. My aim is not to survey the entire field but rather to focus on a narrowly chosen class of disordered systems — Ising models whose nonzero (pair) interactions are i.i.d. random variables. For these systems, I concentrate entirely on equilibrium statistical mechanics; there is no discussion of dynamics.

The two main types of systems considered here are disordered ferromagnets and spin glasses, with the latter both more interesting and more difficult. The emphasis is on questions concerning the number of equilibrium phases of such systems, i.e., the number of ground states (at zero temperature) or the number of extremal Gibbs states (at nonzero temperature).

A recurring theme in the first three chapters of these notes is that these questions about disordered Ising models are often closely connected to interesting issues concerning related percolation models. The percolation models that arise in this way are first-passage percolation (and related random minimal surfaces in higher dimensions), invasion percolation and standard Bernoulli bond percolation (along with its close relative, the dependent percolation of Fortuin-Kasteleyn random cluster models).

The fourth chapter of these notes concerns the low temperature behavior of the Edwards-Anderson model, a paradigm short-range spin glass. The question discussed here is whether and in what sense Parisi's analysis of the Sherrington-Kirkpatrick spin glass, a paradigm mean-field (or "infinite-range") model, is relevant to short-range spin glasses. Closely related is the more general conceptual issue of how to approach the thermodynamic (i.e., infinite volume) limit in systems which may have many complex competing phases. This issue has been addressed in recent joint work with Dan Stein and the main purpose of Chapter 4 is to give a mathematically coherent presentation of our approach.

These notes have benefitted from the comments of many people. In particular, Aernout van Enter read through much of the manuscript and suggested a number of improvements and Alain Sznitman raised the issue of measurability treated in

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New York City

March 4, 1997