Science Networks · Historical Studies
Volume 20

Edited by Erwin Hiebert and Hans Wussing

Editorial Board:

K. Anderson, Aarhus
D. Barkan, Pasadena
H.J.M. Bos, Utrecht
U. Bottazzini, Roma
J.Z. Buchwald, Cambridge, Mass.
K. Chemla, Paris
S.S. Demidov, Moskva
E.A. Fellmann, Basel
M. Folkerts, München
P. Galison, Cambridge, Mass.
I. Grattan-Guinness, London
J. Gray, Milton Keynes

R. Halleux, Liège
S. Hildebrandt, Bonn
E. Knobloch, Berlin
D. Laugwitz, Mühlthal
Ch. Meinel, Regensburg
J. Peiffer, Paris
W. Purkert, Leipzig
D. Rowe, Mainz
A.I. Sabra, Cambridge, Mass.
E. Scholz, Wupperthal
R.H. Stuewer, Minneapolis
V.P. Vizgin, Moskva

Springer Basel AG

George Boole

Selected Manuscripts on Logic and its Philosophy

Editors:

Ivor Grattan-Guinness

Gérard Bornet

Springer Basel AG

Editors:

Ivor Grattan-Guinness
Professor of the History of
Mathematics and Logic
Middlesex University
Queensway
Enfield
Middlesex EN3 4SF
Great Britain

Dr. phil.-hist. Gérard Bornet
Philosophisches Seminar
Université Miséricorde
1700 Fribourg
Switzerland

Published with support of the Swiss National Science Foundation.

Library of Congress Cataloging-in-Publication Data

Boole, George. 1815–1864

George Boole : selected manuscripts on logic and its philosophy /
[edited by] Ivor Grattan-Guinness, Gérard Bornet.

-- (Science networks historical studies ; v. 20)

Includes bibliographical references and index.

ISBN 978-3-0348-9805-8 ISBN 978-3-0348-8859-2 (eBook)

DOI 10.1007/978-3-0348-8859-2

1. Logic, Symbolic and mathematical. 2. Mathematics--philosophy. 3. Boole,
George, 1815–1864. I. Grattan-Guinness, I. II. Bornet, Gérard.

III. Title. IV. Series.

QA9.2.B68 1997

511.3--dc21

Deutsche Bibliothek Cataloging-in-Publication Data

**George Boole – selected manuscripts on logic and its
philosophy** / Ivor Grattan-Guinness/Gérard Bornet. - Basel ;
Boston ; Berlin : Birkhäuser, 1997

(Science networks ; Vol. 20)

ISBN 978-3-0348-9805-8

NE: Grattan-Guinness, Ivor; Bornet, Gérard; GT

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. For any kind of use, permission of the copyright owner must be obtained.

© 1997 Springer Basel AG

Originally published by Birkhäuser Verlag in 1997

Softcover reprint of the hardcover 1st edition 1997

Cover Design: Micha Lotrovsky, Therwil, Switzerland

Printed on acid-free paper produced from chlorine-free pulp. TCF ∞

ISBN 978-3-0348-9805-8

9 8 7 6 5 4 3 2 1

Contents

Editors' Introduction

Part 1: Boole's quest for the foundations of his logic xiii

Ivor Grattan-Guinness

§1 Our purpose	xiii
§2 Life and career	xiv
§3 The fate of Boole's Nachlass	xviii
§4 Organisation and explanation of the selected manuscripts	xxv
§5 The renaissance of logic in Britain	xxvi
§6 Boole, English algebraist par excellence	xxvii
§7 Logic in Boole's Mathematical analysis of logic	xxviii
§8 Between books, 1848–1854	xxxii
§9 Logic in The laws of thought	xxxv
§10 Logic after The laws of thought	xxxvii
§11 Plans and texts for "The philosophy of logic"	xxxix
§12 Where was Boole's philosophy of mathematics?	xl
§13 Outside Boole's universe: Leibniz, Peacock, Babbage and De Morgan	xliii
§14 Boole's immediate legacy	xlvi

Part 2: Boole's Psychologism as a Reception Problem xlvii

Grard Bornet

§15 Introduction	xlvi
§16 Frege's psychologism criticism	xlvi
§17 Boole's anti-inductivism and the problem of introspection	l
§18 Art versus Science	li
§19 Psychology as science	liii
§20 Boole's structuralism	lv
§21 Uninterpretable symbols	lvii

Part 3: Remarks on dating and editing the manuscripts.....	lviii
§22 Remarks on dating	lix
§23 Remarks on editing	lix
Acknowledgements	lx
Notes	lxii

Part A

The Nature of Logic and the Philosophy of Mathematics

Chapter I

The Nature of Logic	1
Logic is the science of reasoning	1
Of what does reasoning consist?	1
General Considerations	2
Of our Conception of Things	2
On the relations of individuals and classes	2
Of the mode in which the mind combines and modifies its own conceptions	3
Of the nature of Propositions	4
Of the limitation of the subject of discourse	4
Laws of the Mental Operation	5
Comparison of the laws of elective symbols with the laws of arithmetical symbols	7
Consequences of the above analogies	8
Examples for Practice	9
Of the interpretation of 0 and 1 as elective symbols	10
Expression of the class not-men etc.	10

Chapter II

Elementary Treatise on Logic not mathematical including philosophy of mathematical reasoning	13
Of Signs	14
Of Propositions	14
Of the Quantity of Terms	15
Of the Quality of Terms	16
Division	17
Of Some of the Varieties of Propositions	18

Of the Limitation of Propositions	20
Of the Conversion of Propositions	21
Of Syllogism	23
Conditions and Rules of Syllogistic Inference	25
General Remarks on Primary Propositions	29
Of Secondary Propositions	29
Of the Demonstrations of Geometry	32
Of the Foundations of Algebra	36
Of the Differential Calculus	38
The real nature of mathematical analysis	41
Chapter III	
[Extracts from a notebook]	42
Miscellaneous Observations on Logic	42
Of Inverse Operations	45
Introduction	46
Of signs	47

Part B

The Philosophical Interpretation of a Theory of Logic

Chapter IV	
Prolegomena	51
Results of the Analysis of Conception	54
Chapter V	
On the Foundations of the Mathematical Theory of Logic and on the Philosophical Interpretation of Its Methods and Processes	63
Analysis of the Operations of Conception as Exercised within the Sphere of Formal Logic	74
Operations founded on Extension	74
Operations founded on Intension	75
Laws of Conception	76
Addition	76
Subtraction	77
Composition	78
Abstraction	79
Law of mixed operations of Addition and Composition	80

Analysis of Judgment as Exercised within the Sphere of Formal Logic	81
Analysis of Reasoning as Exercised within the Sphere of Formal Logic	83
On Systems of Notation	86
Symbolical expression of the formal laws of Logic	88
Analysis of Reasoning resumed	95
Chapter VI	
[Preparatory Notes]	105
Of the nature and scope of Logic as considered a Science	105
Definitions of Conception, Judgment and Reasoning	106
[Conception]	106
Judgment	108
Reasoning	108
[Further Considerations]	110
Chapter VII	
General Summary	112
Theory of Formal Logic	112

Part C

“The Philosophy of Logic” – A Sequel to “The Laws of Thought”

Chapter VIII	
[Preface]	119
Chapter IX	
Table of Contents	121
Chapter I: Of the ordinary Logic	121
Chapter II: Of the operations of thought in relation to the science of number	122
Chapter III: Of the Laws of Thought in Logic	122
Chapter IV: Interpretation of Methods	122
Chapter X	
The Philosophy of Reasoning	123

Chapter XI	
Logic	126
Nature and Office of Signs	129
Methods in Logic	132
The Aristotelian Logic	133
Sir W. Hamilton's Theory of Syllogism	136
The Theory of Professor De Morgan	138
Of the Ultimate Laws of Thought	141
Of the Method of This Work	143
The Method of Algebra	144
Method of This Work	148
Results of This Method	150

Part D

Miscellaneous Matters, Letters and Fragments

Chapter XII	
On Belief in Its Relation to the Understanding.....	157
Chapter XIII	
The Philosophical Idea of Freedom	162
Infinite evolution	162
Chapter XIV	
Note [to Aristotle]	164
Chapter XV	
Philosophy of Mathematics	167
Chapter II: On Geometry	171
Chapter XVI	
[Various fragments, apparently late]	179
[Fragment 1]	179
[Fragment 2]	180
Digression on the Nature of Algebra	180
[Fragment 3]	183
Principles of Expression	183
Laws of the Symbols	184
[Fragment 4]	185

[Fragment 5]	187
[Fragment 6]	188
[Fragment 7]	189
Chapter XVII	
Letters to Cayley, Lubbock and Penrose	191
Arthur Cayley	191
Cayley to Boole, 2 December 1847	191
Boole to Cayley, 6 December 1847	191
Cayley to Boole, 7 December 1847	192
Boole to Cayley, 8 December 1847	193
Boole to Cayley, 10 December 1847	194
Cayley to Boole, 11 December 1847	195
Boole to Cayley, 14 December 1847	196
John William Lubbock	197
Boole to Lubbock, 16 February 1849	197
Boole to Lubbock, 22 February 1849	198
John Penrose	200
Boole to Penrose, 13 March 1855	200
Textual Notes	203
Explanation of the description of the material	203
The two Signature systems "MM" and "RS"	203
The Boole/Falk List of 1896	204
Details on individual documents	206
Notes on the individual chapters	208
Chapter I: The Nature of Logic	208
Chapter II: Elementary Treatise on Logic	208
Chapter III: [Extracts from a notebook]	209
Chapter IV: Prolegomena	209
Chapter V: On the Foundations of Logic	210
Chapter VI: [Preparatory Notes]	211
Chapter VII: General Summary	212
Chapter VIII: [Preface]	212
Chapter IX: Table of Contents	213
Chapter X: The Philosophy of Reasoning	213
Chapter XI: Logic	213
Chapter XII: On Belief in Its Relation to the Understanding	215
Chapter XIII: The Philosophical Idea of Freedom	215
Chapter XIV: Note [to Aristotle]	216
Chapter XV: Philosophy of Mathematics	216
Chapter XVI: [Various fragments, apparently late]	217
Chapter XVII: Letters to Cayley, Lubbock and Penrose	220

Bibliography	222
Index of Names	229
Index of Subjects	231

Editors' Introduction

Part 1: Boole's quest for the foundations of his logic

Ivor Grattan-Guinness

§1 Our purpose

The adjective “boolean”, sometimes even “Boolean”, is known to mathematicians for a powerful algebra, to electrical engineers and computing specialists who apply it widely in parts of their work, to philosophers as a component of logic, and to historically-minded people as one George Boole (1815-1864), who formed an algebra of logic from which we enjoy these modern fruits. This book publishes some of the manuscripts in which he tried but failed to find a satisfying philosophy to underpin that logic, and conform with the elements of philosophy and psychology which he felt himself to have addressed; almost all appear for the first time.

After a short recollection of Boole's life and work (§2), this introduction continues with the story of the manuscripts, in particular of the collection from which our edition is drawn (§3); the organisation of our selection is described in §4. Then, following brief surveys of the revival of interest in logic in Britain from the 1820s (§5) and of Boole's mathematical background (§6), we describe the main phases of his logical career. He wrote his first book, *A mathematical analysis of logic* (hereafter “*MAL*”) of 1847 (§6-7). Then a period of rethinking over seven years (§8) culminated in his second and more substantial book, *An investigation of the laws of thought* (“*LT*”) of 1854, which included applications of his logic to probability theory (§9). After that he wrote many manuscripts, especially to write a third book on “The Philosophy of Logic” (“*PL*”) (§10-12). We end with some remarks on four figures who did *not* influence Boole as one might expect (§13), and a short

review of the impact of his publications up to the end of the century (§14). Throughout we give more space to his mathematical details and procedures, because he tended to assume them as known in the manuscripts, where he covered the philosophical and psychological aspects much more amply.

§2 Life and career

George Boole was born on 2 November 1815 in Lincoln, the eldest of four children of a local tradesman John Boole (1777-1848) and his wife Mary Ann (1780-1854).¹ He adopted a love of learning from his father to such an extent that already in his teens he taught at schools in Lincoln and nearby; in his 20th year he even opened his own school. He also learnt classical and modern languages and wrote poetry, and taught himself mathematics (and also French and German) to a level which permitted him to engage in research in mathematics. He began publishing in 1841, first and frequently in *The Cambridge Mathematical Journal*, which had been founded two years earlier by D. F. Gregory (1813-1844), his initial mathematical mentor and inspiration (§7).

Although this early work gained considerable attention, Boole had to continue in school-teaching until the Queen's University was proposed in 1845, as a secular institution for higher education in Ireland. After delays caused by the potato famine, and Catholic opposition, Queen Victoria formally opened the constituent colleges, including Queen's College, Cork, in 1849, and Boole took up then his post as the founder Professor of Mathematics. While remembered as a conscientious and at times inspiring teacher, he was not able to find any distinguished followers among his student clientele. Never really happy with life in Ireland, he was recognised only locally; in particular, he had very little contact with W. R. Hamilton (1805-1865) up north in Dublin, the only other mathematician in the island of comparable calibre, even though they were both pioneering new algebras. Further, the Royal Irish Academy did not elect him to their throng. However, he received honorary doctorates from Trinity College Dublin in 1851 and Oxford University eight years later, and was made Fellow of the Royal Society in 1857. For likenesses there survive a very few sketches, and one photograph taken in London probably in 1860 and often reproduced; when printed the correct way round he is looking to the viewer's left (Figure 1).

Throughout his career Boole published steadily, in two areas which were tightly linked, as we shall see in §6 and §7. He started with the differential and integral calculus, where he became a major figure in certain algebraic theories popular in England and Ireland. He also wrote two well-received textbooks: *A Treatise on Differential Equations* (1859a in the bibliography),

Figure 1: The well-known photograph of Boole, taken in London 1864 (Reproduction D. MacHale)

and *A Treatise on the Calculus of Finite Differences* in the following year (1860a).

Boole's other area was logic, which he publicised principally in *MAL* (1847a) and *LT* (1854a), and in some papers. Today the latter book is recognised as more authoritative, but at the time it gained less attention than *MAL*. To accentuate the historical irony, in his lifetime and for years afterwards his work in logic was much *less* well recognised than were his contributions to the calculus, where his methods and especially textbooks were used. Doubtless aware of the apathy over the logic, he attempted at various times, especially soon after the publication of *LT*, to write the non-technical PL.

In 1855 Boole married Mary Everest (1832-1916), daughter of the Reverend Thomas Everest and niece of Sir George Everest, after whom the famous mountain is named. They produced five daughters, four of whom had varied and distinguished careers of their own: two mathematicians, a chemist and a novelist. One of them will appear in §3 as an archivist. Mary assisted her husband with his two textbooks on differential equations, as the first reader for intelligibility and also as checker of some of the exercises. After his death she proselytised Boole's philosophy, especially its application to educational questions, both in teaching and in print. Although she was regarded as a crank, her understanding of his ideas on logic and education were basically sound (Laita 1980a): scattered around her rather verbose collected works are some valuable hints and data. In particular, the title of the short piece "Mount Carmel in London", and its initial publication in the journal *The Crank* in 1904 (Boole, M. 1931a, 75-79), should not deter readers from its merit.

Like his wife, Boole was deeply religious. According to her, when a youth he came to the idea that a universe was central to knowledge and revelation as in "the Great All stressed as a unity in the Scriptures", and monism in the Jewish faith (Boole, M. 1931a, 473, 951-952), and thereafter he affirmed ecumenism. He implicitly exhibited his position in *LT* by devoting ch. 13 to sophisticated logical analyses of propositions due to Samuel Clarke and Benedict Spinoza concerning the necessary existence of "*Some one unchangeable and independent Being*" (p. 192). However, he was a Dissenter, and at a time when such movements were gaining considerable ground in Britain against strong resistance from the established Trinitarian Church of England. He alluded to his stand in print very discreetly a few lines from the end of *LT*, where he mentioned "the Father of Lights" and finished off with some enigmatic lines about the bearing of religious belief upon his logic.

Ecumenism was very important to the philosophy of Boole's logic; for its Universe 1 symbolised the Godhead standing over and lighting the factions into which the Christian Church was split. He was especially admiring of Frederick Denison Maurice (1805-1872), who advocated ecumenism with great force in mid century and so was dismissed from his chair at King's

Figure 2: Boole's house at Ballintemple, where he died
(Photograph I. Grattan-Guinness)

College London. Boole spent parts of several summer vacations in England in his last years, studying in London libraries and also attending Maurice's Sunday services.

In similar reticence, Boole kept his political views to himself: he offered no utterances on the condition of the Ireland in which he lived after the famine. But in a manuscript discussion of words, written in the revolutionary year 1848, he embodied his liberal aspirations (page 5 below).

Boole died as a result of foolishly walking the three miles from home to university without proper protection from a rain storm, so probable an event in Cork. The ensuing pleuro-pneumonia took him away in his 50th year on 8 December 1864; he was buried at Blackrock, near Cork and the family residence in Ballintemple (Figures 2 and 3).

The popular *Illustrated London News* published a short obituary by the Vice-President of the College (Ryall 1865a) together with a fine sketch (Figure 4). Boole's former friend the teacher and mathematician Robert Harley (1828-1910) published a fine long eulogy 1866a, and condensed from it a routine obituary 1867a for the Royal Society. The Scottish educationalist and Shakespeare scholar Samuel Neil (1825-1901) also wrote an excellent piece 1865a in his own journal, *The British Controversialist*.² In Canada there

Figure 3:
Grave of Boole at Blackrock,
with his biographer D. MacHale
(Photograph I. Grattan-Guinness)

appeared a lengthy appreciation of *LT* (Young 1865a), together with a short notice (Anonymous 1865a). But no obituaries were written by leading mathematicians.

In addition, and exceptionally for a mathematician, two stain-glass windows were erected in Boole's memory. One was inserted around 1867 in the Great Hall of Queen's College Cork; he is shown in one of the ten panels (Figure 5) portraying various world luminaries. The other was inserted into Lincoln Cathedral two years later, thanks to a subscription raised by fellow townspeople.

§3 The fate of Boole's Nachlass

After Boole's death and burial, his widow moved the family to London, where Maurice found her a post at his Queen's College for women's education in Harley Street. Although the government awarded her a Civil List pension of £100 per year, lack of money presumably forced her to sell his library. No

Figure 4:
Engraving of Boole,
published with Ryall's
obituary 1865a

list of it survives, but it was probably not large; it seems likely that most of his wide reading was done in libraries, or at least thanks to their holdings.

As well as teaching and applying George's ideas to education, Mary began sorting his manuscripts with the help of her third daughter Alicia (1860-1940). Mathematical colleagues saw them: William Spottiswoode (1825-1883) fulfilled her request to send some of the mathematical ones to the Royal Society, and Isaac Todhunter (1820-1884) quickly formed those which Boole had prepared for a second edition of the book on differential equations into a "supplementary volume" of the (Boole 1865a), the edition itself was basically just a corrected reprint.

In addition, Augustus De Morgan (1806-1871), Boole's most eminent contemporary in the algebra of logic (§13) and a regular correspondent, looked through the logical manuscripts. He felt in his report 1867a that only one was fit for printing: Boole 1868a on numerically definite syllogisms (a topic initiated by De Morgan himself in his 1847a, ch. 8), seemingly written around 1850.

Mrs Boole and Alicia sorted out the mathematical from the logical manuscripts, organised each part into lettered series, and transcribed by hand many non-symbolic logical texts and also letters; they also gathered

together a group of notebooks. Eventually she arranged to give them (hereafter "the collection") to the Royal Society in 1873. However, family interest in them had not ceased. In particular, in the 1880s Alicia, then living in or near Liverpool, teamed up with fellow townsman H. J. Falk, mathematician by inclination and barrister by profession. They formed a small group to study Boole's work; it included an actuary called Walter Stott, whom Alicia married in 1890. In order to prepare a new edition of *LT*, which had become very rare, they asked to borrow back the collection from the Royal Society in 1889. The Council minutes of 20 June duly record that "Leave was granted to Miss Boole to borrow the MSS. of the late Prof. Boole, and to make extracts from the same" (Royal Society 1893a, 262).

Alicia and her helpers looked in detail at the logical part; they tried to determine which manuscripts were written before and which after *LT*, and to pick out those of the latter destined for the succeeding *PL*. They organised the collection further into various lettered and numbered sections, and prepared typescripts of several logical manuscripts and some letters. They returned it to the Royal Society in 1896; Council minutes of 29 October (Royal Society 1899a, 293-294) record that

The Boole MSS. presented to the Society by the widow of the late Prof. George Boole and lent to Miss Boole in 1889, for the purpose of making extract therefrom, have now been returned, the whole of the MSS having been collated and arranged in order, and type-written copies of several of them furnished for the Society's Library by Mr. H. J. Falk, on behalf of the Boole family. It is proposed to proceed now with the binding of the collection.

The Society also resolved to thank Falk "for the trouble he has taken in arranging and copying the Boole manuscripts". Mercifully, the binding was never effected; in addition, no publications came from all this effort, although Mrs Boole still published on (her version of) his theories from time to time.³

Then, in 1905, enter Bertrand Russell (1872-1970), surprisingly and probably also surprised. In his early thirties, he was deeply engaged in a vast logical enterprise with A. N. Whitehead (1862-1947) of a quite different kind from Boole's (§14) which was to yield their three-volume *Principia mathematica* (1910-1913). Perhaps not appreciating the great differences between this and her husband's enterprises, Mrs. Boole seems to have thought of Russell as a suitable editor. Once again nothing eventuated; but parts of the related correspondence survives in Russell's *Nachlass*. Falk's son Oswald Toynbee⁴ told Russell on 18 February 1905, not quite accurately, that

Mrs. Boole after George Boole's death gave a large collection of his unpublished papers to the Royal Society absolutely. The more strictly mathematical portion of the papers was looked into by Todhunter and I believe De Morgan, the result of their work being

Figure 5:
Panel showing Boole in the
Memorial Window in the Great
Hall of University College, Cork
(Photograph I. Grattan-Guinness)

the supplementary volume of the Differential Equations. My father thinks that they extracted nearly all that was valuable of this nature.

There were however in addition many papers which appeared to be connected with "The Laws of Thought", and with the permission of the Royal Society my father had all the papers out for some years, and in conjunction with Alice Boole (Mrs. Stott) he produced some sort of order in the logical portion of the papers. They are I believe the beginnings of a book designed to render the ideas of the "Laws of Thought" more intelligible to the non-mathematical reader. The papers after arrangement were copied, (3 typewritten copies were made of which my father has one) and the originals with a copy given back to the Royal Society where they now lie. The Royal Society asked my father if he had any recommendation to make re publishing, but apparently he did not like to judge for himself, and no one else has taken the matter in hand. The papers may be worth publishing and apparently all that is required is the judgement of somebody qualified to decide in such a matter. If you would like to see the papers (or have not already seen them) my father would be glad to send you his copy after obtaining Mrs. Boole's permission.

The old lady is, I believe, rather a difficult person to deal with but I expect there would be no opposition.

You will see that I rather overestimated the work which remains to be done. I had heard my father talk of the confusion of the papers but did not know that anything had been done to put them in order or that they had been restored to the Royal Society.

Russell corresponded with his friend the logician Louis Couturat (1868-1914) about the request at this time, correctly considering him to be a far more appropriate candidate as editor; among other work, Couturat had already written on the similarities between Boole's and Leibniz's logics (§13) (1901a, 344-354, 385-387). On 7 February he wrote to Russell, regarding it as "a great honour for me to be charged with the edition of these manuscripts, on condition that they are worth the effort (that is to say that they are not simple drafts or copies of what Boole had published) and on condition that they do not pass my competence such as treating probability theory."⁵ In reply on 9 February, Russell offered to translate into English any editorial material that Couturat might write.

However, after hearing nothing from Falk, he wrote on 13 October from his home at Bagley Wood near Oxford:

I have never heard anything further from you about the Boole MSS in your father's possession. As it occurred to me that perhaps you were kept from writing by not knowing my address, I am writing to say that the above is my permanent address. I heard from M. Couturat that if the MSS proved interesting he would probably undertake to edit them; & I feel no doubt that he is the most suitable person living for the purpose.

This initiative inspired further recollections by H. J. Falk to his son on 18 October, which were forwarded to Russell:

My dear Oswald,

It would be well to make clear to Mr. Russell that the Boole MSS are not in my possession or control. They are at the Royal Society to whom they were committed by Mrs. Boole on her husband's death 40 years ago.

Any control I have is solely by courtesy through my having (with Mrs. Boole's consent and permission of the Royal Society Council) collated and arranged the chaotic mass of papers. I then had the MSS typed, presented one copy to the Royal Society and retained one. I have for 20 years wished to see the "Laws of Thought", reprinted with some of these MSS and perhaps a biography added, and would gladly give Mr. Russell and his friend any assistance of which I am capable to that end. It would be a real public service and I regret that there are objections in the family, which so far I have

never overcome. I feel bound to respect them, not merely on friendly grounds. Mrs. Boole holds various metaphysical views as to Boole's Logic, based chiefly on Gratry,⁶ which I have never been quite able to follow. But I cannot venture for that reason to conclude that they are without foundation.

She must remember much of her husband's thought, which no one now can know. These views are not shared by her very able daughters and especially not by Mrs. Stott, who has most of her father's mathematical and logical genius. Now Mrs. Stott worked very hard with me on the MSS, and I cannot disregard her wishes. To put it briefly, her fear (in which the others concur) is that, if the L.T. or M.S.S. be printed by anyone, Mrs. Boole will not only disown any connection with the new edition but will attack it as an inadequate presentation of Boole's thought. Hence they would prefer nothing done during the old lady's life. A few words as to the M.S.S. — In the ten years between the L.T. (1854) and his death (1865) Boole was continually working at a sequel to it. The important part of the MSS consists of those efforts. His great mind sought to lift the Principles and Methods of Thought even further than De Morgan and Jevons⁷ seemed to see, — into a clear region beyond the schoolmen and formalists, whether logical or mathematical. Thus the beauty and power of those MSS are, if I am any judge, too precious to be buried for ever.

Russell responded to this envoi on 25 October as follows:

Many thanks for your letter, and for the letter from your father enclosed. It would give me the greatest pleasure to meet your father if it can be arranged. I hope if anything brings him to Oxford, he will come here at any time that suits him; otherwise, we might meet in London.

From his letter I see that the objections to printing during Mrs. Boole's lifetime are very strong. I have always taken it for granted that her views of Boole's opinions must be wrong, particularly since I read Gratry about a year ago: Gratry seemed to me not nearly good enough. I should therefore be quite in sympathy with the feeling that it would be better to wait.

I am afraid from your father's letter that he thinks I am willing to edit the MSS myself, in case at any time they are to be edited. This is hardly the case. I have at all times a great deal of original work that I want to do, & I should hardly be willing to give up so much time. But M. Couturat is more learned than I am: he has done an *admirable* edition of Leibniz's unpublished MSS on Symbolic Logic [Couturat 1903a]; he is willing to undertake the task and is far better qualified for it than I should be. I should of course be glad to assist him in any way, particularly as regards language. But *he* would be the editor.

Despite Couturat's willingness to act, nothing happened in the end. However, a few years later the mathematician and historian of mathematics Philip Jourdain (1879-1919), who had taken Russell's course in mathematical logic while an undergraduate at Cambridge in 1901-1902, became interested in Boole, as part of a general study of the recent development of symbolic logic. He wrote an article 1910a on Boole, borrowing from the Stotts copies of various materials, and also from Boole's former friend the mathematician and teacher Robert Harley (1828-1910) a bound volume containing Boole's letters to him and copies of various others; sadly, this volume has never been seen since. Jourdain's only other act was to publish with the Open Court Company in 1916 a new edition of *LT*; however, he contributed merely a two-page introduction, corrections of errata, and a (very welcome) index, and did not include any manuscripts. Published as "Volume 2", it was intended to be preceded by an edition of Boole's papers on logic; but he seemed not to have started on this project before his early death. His manuscripts were passed by his widow to A. E. Heath and J. M. Child as literary executors (Rhees 1955a); but most have disappeared.

Thus ended serious Boole scholarship until the late 1940s. The centenary of *MAL* in 1948 stimulated a survey Kneale 1948a of Boole's life and work in *Mind*; and a photoreprint of the book that year aroused some interest, such as an article Prior 1949a on propositions in Boole. Then four years later two British philosophers used the collection. Mary Hesse quoted a few passages in an article 1952a; and at the suggestion of Jourdain's literary executor A. E. Heath, Rush Rhees produced Jourdain's missing volume with a selected edition Boole 1952a of published and unpublished writings on logic and probability, prefaced by his own informative "Note in editing". (Jourdain's edition of *LT* was reprinted with it, for the second time.) Both these works contain short quotations from some of the manuscripts included here. In 1954 the Royal Irish Academy celebrated the centenary of the publication of *LT* with a special meeting and proceedings; Rhees 1955a and Feys 1955a will be cited from the latter.

After another pause, this time of 20 years, several historians used the collection. Susan Wood worked on it, in the course of preparation of a doctoral dissertation 1976a on Boole's theory of propositional forms. The most noteworthy later results include Desmond MacHale's biography 1985a, and Maria Panteki's exploration of the connection between algebras and logic in Boole in her doctoral thesis 1992a on this theme in Britain over the period 1810-1870.

The collection is currently stored in nine boxes, together with a separate set of notebooks. Some general remarks on our editing and dating are given in the short set of "Remarks" following this introduction, with the necessary minimum of grim details supplied in the "Textual notes" which follow the transcriptions.

As with many figures, Boole's full *Nachlass* has become somewhat scattered. In particular, quite a few documents found their way to the USA, seemingly in the family of Boole's elder daughter Mary Ellen (1856-19??), and were sold at auction to Cork University Library in 1983. They include an unpublished biography by his sister Mary, quite a few letters and notebooks, and some material from his school-teacher days. While we have consulted them, none has been included in our selection.

§4 Organisation and explanation of the selected manuscripts

Boole seems to have tried again and again to articulate his philosophy of logic, usually starting each time from scratch (as he told De Morgan in a letter quoted in §11). Only rarely did he produce a substantial text; quite often the outcome was more notes or another work-plan. He never dated a manuscript, although occasionally he indicated a time by reference to *MAL* or *LT*; and his handwriting did not noticeably change over the years. Another complication is that sometimes he wrote essays or plans on logic on the versos of discarded folios of mathematical remarks or calculations, and some of these sequences have been ordered for mathematical rather than logical continuity.

Thus our selection has been made from a scattered and chronologically unclear textual universe. It comprises around 40% of Boole's unpublished manuscripts on logic. We hope that it provides a wide and characteristic cross-section of his meditations; it complements a few manuscripts which were included by Rhees in his edition Boole 1952*a*. We have divided it into four Parts lettered A-D, and also into Chapters numbered I-XVII. Most Chapters comprise one integral manuscript, and some contain Boole's own chapters or at least references to them; so to avoid confusion we refer to his (and anyone else's) "chapter(s)". In the same spirit, this book has "page(s)" while all others have "p(p)". Unless indicated by our square brackets, the title of each Chapter is Boole's.

The orders of, and in, Parts A-C are hopefully chronological, at least coarsely so. There seems to be nothing before *MAL*; Part A is placed between *MAL* and *LT*, and is surveyed in §8. Part B contains manuscripts written after *LT* (§10), and Part C those which seem to have been specifically intended for the third book (PL) on the philosophy of logic (§11); the division between Parts follows the intention of Alicia Boole and Falk a century ago, but we cannot claim it to be perfect. Those sections of all these writings relating most closely to mathematics are highlighted in §12, and from them a conjecture is made about his failure to produce PL. Part D contains two

manuscripts of uncertain date (Chapters XII and XV) whose contents place them rather outside Parts B and C, some interesting notes and undated fragments, and a few letters; they are all noted in §8 and §10-§12 as appropriate.

After the "Textual Notes" comes the bibliography, which combines the items cited in this introduction, a few other historical writings of note which have not been cited, and the works mentioned or alluded to in the manuscripts and identified in the Notes. An index of names and subjects completes the edition.

§5 The renaissance of logic in Britain

Boole's contributions to logic came at a time when the discipline was experiencing a remarkable revival after a long period of quietude. While he seems not to have been profoundly influenced by recent developments when forming his own approach, he drew upon them, especially in his manuscripts. Some main details, therefore, should be noted.

The study of logic in Britain was then in a peculiar state. The classical tradition, based upon inference in syllogistic logic, was still in place. But for a long time an alternative tradition had been developing; inspired by John Locke (1632-1704) and continued in some ways by the Scottish Common-Sense philosophers of the late 18th century. Critical of syllogistic logic, especially for its narrow concern with inference, adherents of the new tradition sought a broader foundation for logic in the facultative capacity of reasoning in man, and included topics such as truth and induction which we might now assign to the philosophy of science. Showing more sympathy to the role of language in logic than had normally obtained among the syllogists, they laid emphasis on signs as keys to logical knowledge; indeed, Locke used the word "semiotics" in the fourth Book of his *Essay concerning human understanding* (1690) (Buickerood 1985a). At the cost of some simplification, we shall refer to this approach as "the sign tradition".

The principal source of this revival of interest can be precisely identified: the *Elements of logic* (1826) by Richard Whately (1787-1863), based upon his long essay on the topic in the *Encyclopaedia metropolitana*, which had been published in two parts in 1823. In the book version he surveyed syllogistic logic in a "Synthetical Compendium" of nearly 80 pages, and used it in a following chapter of equal length on "Fallacies". But he was much taken with the sign tradition, endorsing some of its criticisms of syllogistic logic in his "Introduction" and opening chapter.

Whately's book prompted an astonishing response; he soon produced revised editions (the fourth in 1832), and further ones later. Several figures wrote lengthy reviews or commentaries, or used it in writing textbooks for

university use. All figures in logic of that time drew something from him, whether positively or negatively, at least in English-speaking countries.⁸ Boole was one reader; so we use Whately as our main source for several issues, although other authors doubtless also influenced him.⁹

The main single innovation was made by George Bentham (1800-1884), in his *Outline of a new system of logic with a critical examination of Dr Whately's "Elements of Logic"* (1827); but it sold very poorly, and so was not widely recognised until the 1850s. He greatly increased the number of forms in syllogistic logic by adding to the standard ones such as "All As are Bs" the types "All/Some/No As are All/Some/No Bs". The name "quantification of the predicate" became attached to this extension during the 1840s (it is the origin of the word "quantification" in logic). A priority row over its genesis was initiated in the mid 1840s by the Scottish philosopher Sir William Hamilton (1788-1856) (his approach to logic is summarised in §11). However, although he had seen Bentham's book, his target was De Morgan (Laita 1980a). It was this contretemps which inspired Boole to write up his ideas on logic, and to publish them as *MAL* in 1847.

§6 Boole, English algebraist par excellence

Boole's contributions to mathematics and logic were both characteristic of and important for the development of English mathematics in his time; for they were entirely guided by algebras of new kinds, which we consider here. His earliest work extended the notion of the invariant, which was to become a major English industry in the hands of Arthur Cayley (1821-1895) and J. J. Sylvester (1814-1897) (Parshall 1989a). But his main concern was drawn from the algebraic version of the calculus based upon Taylor series, proposed by J. L. Lagrange (1736-1813), extended in the early 19th century by the Alsatian L. F. A. Arbogast (1759-1803), and continued by a few minor French mathematicians (Grattan-Guinness 1990a, chs. 3-4).

In this version the operation of differentiating a mathematical function $f(x)$ was represented by the letter "D", so that the derivative $f'(x)$ (to use Lagrange's notation) was written " $Df(x)$ ". An algebra was developed in which second differentiation was written as the power " D^2 ", the inverse operation of integration was " D^{-1} ", and so on. The resulting theory was used principally to solve differential and difference equations, sum series and evaluate integrals. The results were often correct although the foundations of this algebra were mysterious; for example, as critics pointed out, the orders of differentiation were identified with the corresponding powers of the first ($d^n y/dx^n = (dy/dx)^n$ for $n \geq 2$).

Inspired partly by Gregory, Boole became a leading practitioner of this theory, not only making new applications but also seeking to explicate the

foundations and make the theory more rigorous (Panteki 1992a, ch. 4). His principal paper, "On a general method in analysis", was submitted to the Royal Society; after wishing to decline it, they published it in their *Philosophical Transactions* as Boole 1844a, and then gave him their Gold Medal for it. In this paper and related ones, he explored the D-methods to new lengths, applying them to both ordinary and partial differential equations, and examining non-commutative operator functions F and G for which $F(D)G(D) \neq G(D)F(D)$. He seems to have hoped that one general method could be found; but his own work revealed the hope as overly optimistic.

Three years later *MAL* appeared, in which Boole gave the first version of his logic. As its title shows, he saw his "mathematical analysis of logic" as *applied mathematics*, a view which he always maintained. In this application he worked with mental acts (called "elective symbols", in the manner common to algebraists of that time of identifying the symbol with the object or operation symbolised); the act of selecting, say, men out of the Universe of humans. He also spoke of the class of men so selected, and interpreted his theory for both operations (mental acts) and objects (classes). "Let us employ the letters X, Y, Z, to represent the individual members of classes [and] The symbol x operating upon any subject comprehending individuals or classes, shall be supposed to select from that subject all the Xs that it contains" (*MAL*, 15). The "product xy will represent, in succession, the selection of the class Y, and the selection from the class Y of such individuals of the class X as are contained in it, the result being the class whose members are both Xs and Ys" (p. 16). "The equation $y = z$ implies that the classes Y and Z are equivalent, member for member" (p. 19).

Boole then set down the basic laws with these mental acts obeyed, in a form closely similar to those for the differential operators in 1844a (*MAL*, 17-18). Writing π and r for functions of D operating on the mathematical functions q and r , and u , v , x and y for elective symbols, he named and wrote the laws for the two theories as follows;¹⁰ l , m and n are positive integers, with $n \geq 2$:

$$\pi r q = r \pi q \quad \text{commutative law} \quad xy = yx \quad (6.1)$$

$$\pi(q+r) = \pi q + \pi r \quad \text{distributive law} \quad x(u+v) = xu + xv \quad (6.2)$$

$$\pi^l \pi^m q = \pi^{l+m} q \quad \text{index law} \quad x^n = x \quad (6.3)$$

§7 Logic in Boole's *Mathematical analysis of logic*

Boole's principal single innovation was law (6.3)₂ for logic: it stated that the mental acts of choosing the property x and choosing x again and again is the same as choosing x once, or that the class x taken together with the class

Figure 6:
Painting of the young Boole in
his early thirties (around 1845)
(Reproduction D. MacHale)

x over and over again gives the class x . (The possible parentage of this law in Leibniz is considered in §13.) As consequences of it he formed the equations

$$x(1-x) = 0 \quad \text{and} \quad x+(1-x) = 1 \quad (7.1)$$

which for him expressed respectively the law of contradiction and the law of excluded middle. The various symbols were interpreted as follows: for brevity we mention just the readings as classes. “1” symbolised a “Universe” which “comprehend[s] every conceivable class of objects whether existing or not” (p. 15), and also the corresponding operation (p. 20); but this was rather too Universal, since within this 1 true propositions cannot be distinguished from tautologically true ones. “-” represented the subtraction of a class from one of which it was a part; however, since he never used a symbol for “not”, his expression of that law was not complete (Ellis 1863a). “0” denoted “Nothing” — none too clearly, since it was a class of some sort. In addition, the theory of classes which he used was the extensional part-whole kind, with (for example) x as part of 1. He also defined the conjunction ($x+y$) of two classes only when x and y had no parts in common: this met with a poor reception (§14). He left implicit quite a few laws; associativity of addition and of multiplication, for example, and those which 0 and 1 obeyed.

One main purpose of Boole's algebra was to take one or several propositions as premises, express them as algebraic equations, and then to relate a selected elective symbol to the others; the resulting proposition was a logical consequence of the premises. The double interpretation of letters as elective symbols or as classes gives the algebra a feel of model theory, although he cannot be read in its modern terms, or even of its pioneers around 1900 (Scanlan 1991a).

Boole realised that a given set of premises may not have any solutions, or maybe more than one; and to accommodate the latter possibility he introduced "V" to symbolise an indefinite proportion of members of a class, from none through "some" to all of them; the corresponding elective symbol was "v" (p. 21). This notion has often perplexed readers, but it allows him to handle the lack of a cancellation law in his algebra: namely that

$$\text{if } xy = xz, \text{ then it is not necessarily the case that } y = z. \quad (7.2)$$

In addition, it enabled him to express many propositions in terms of equations rather than inequalities, which would have complicated the algebra considerably. However, he offered no laws which "v" should satisfy, and could not always distinguish between traditional forms of proposition and those involved in the quantification of the predicate (which he did not analyse explicitly); for example, " $vx = vy$ " could cover both "Some Xs are Ys" and "Some Xs are some Ys" (pp. 21-22).

Further, contrary to Boole's apparent belief, the solutions found by his methods were not always complete (Corcoran and Wood 1980a). For example, for the universal affirmative proposition "All Ys are Xs", symbolised as

$$(1-x)y = 0, \quad \text{he put forward} \quad y = vx \quad (7.3)$$

as "the most general solution" (MAL, 25); but he should have noticed that $x = 0$ was missing from it, and that it did not hold if $x = 0$ and v was a class such that $vy \neq 0$. It is curious that a logician should seem to slip over the difference between necessary and sufficient conditions (another apparent example arises in the manuscripts at (10.1)), or that a mathematician with a strong interest in singular solutions of differential equations (§12) did not notice analogous problems in his logic. In fact, in this case Boole realised that $v = xy$ was a better version than (7.3)₁: so did the Irish mathematician Charles Graves after reading MAL, and he told the Royal Irish Academy in a note 1850a.

A substantial section of MAL (pp. 48-59) treated "Of Hypotheticals", propositions composed of two or more propositions A and B linked by a connective, such as "A and B", say, or "If A, then B". His procedure followed the practise of Whately and others to treat them as categorical propositions concerning their constituent propositions; but the resulting algebra was rather strange (Prior 1949a, Hailperin 1984a). "1" was now charged to symbolise the "hypothetical Universe" of all circumstances, so that x elected

those for which proposition X was true. Then, for example, “the Proposition, either X is not true, or Y is not true, the members being exclusive” because of the restriction on “+”, came out as

$$y(1-x)+x(1-y) = 1 \quad \text{for} \quad x-2xy+y = 1 \quad (7.4)$$

(p. 53); the index law $(6.3)_2$ converted it to a quadratic equation, and then factorising yielded solutions. This part of his theory did not endure (§9).

In order to generalise his methods of drawing consequences from premises, Boole devoted the rest of his book to various general expansion theorems and processes of elimination of letters. The theorems were proved by an extraordinary use of Taylor's series with $(6.3)_2$ used to get rid of the powers (pp. 62-63); they took these forms for a function “ $\phi(x)$ ” of one mathematical act, and “ $\phi(xy)$ ” or a function of two of them:

$$“\phi(x) = \phi(0) + \{\phi(1) - \phi(0)\}x = \phi(1)x + \phi(0)(1-x),” \quad (7.5)$$

$$“\phi(xy) = \phi(00)(1-x)(1-y) + \phi(01)y(1-x) + \phi(10)x(1-y) + \phi(11)xy.” \quad (7.6)$$

The basic components x and $(1-x)$ of (7.5), or xy , ... and $(1-x)(1-y)$ of (7.6), perform like a basis in a space of expansions. Although vector space theory was not yet explicitly in place, linear combinations were well recognised, and Boole soon stated the generalisation appropriate to his algebra on p. 64:

$$“a_1t_1 + a_2t_2 \dots + a_rt_r = 0” \quad (7.7)$$

for base terms $\{t_j\}$ with coefficients $\{a_j\}$.

Boole obtained the solutions by the usual methods of linear equations adapted to the laws of this algebra. As a result, some of them were expressed as quotients of the functions, with particular values for the coefficients. For example, $(7.3)_1$ was solved for y from (7.5) as

$$“y = \frac{1}{1-0}x + \frac{0}{0-0}(1-x) = x + \frac{0}{0}(1-x)” \quad (7.8)$$

with “ $\frac{0}{0}$ ” was explained as an alternative to “ v ” (p. 74), a quotient notation which was to feature prominently in *LT*. Despite the extended method of solution, the same “complete solution” $(7.3)_2$ was still found. These are the principal mathematical features of Boole's *MAL*, deeply influenced by his work on differential equations.

Philosophically speaking, as the references to “mental acts” shows, Boole espoused a kind of psychologism. This word is much over-used in this context, since most logics affirm some relationship between the mind and its linguistic and logical products. So his sense needs specification.

One form of psychologism sought to reduce all knowledge to actual mental processes; Boole deliberately avoided this form (for example, *MAL*, 16). At the opposite pole is a social psychologism, which ignored the inner workings of the mind and relied upon empirical evidence as philosophical basis. It was best represented at that time by John Stuart Mill (1806-1873),

whose *A system of logic* was first published in 1843, four years before *MAL*. Much of Mill's book would today be regarded as philosophy of science; on logic he asserted empiricism to the length of claiming even that basic laws such as that of excluded middle were formed by inductions from observation (Richards 1980a). Boole cited Mill only once in print, on a neutral matter, in the preface of *MAL*; but he objected to Mill's philosophy in a letter of 1855 (page 200), and avoided this kind of psychologism also.

Boole was psychologistic in that he saw his study as a normative account of the correct use of thinking — or, better, of thoughts, the products of thinking, and without concern over the manner of manufacture. He always stuck to this position. In *MAL* "That which renders Logic possible, is the existence in our minds of general notions", and their "elementary laws upon the existence of which, and upon their capability of exact symbolical expression, the method of the following Essay is founded" (pp. 5-6); *LT* opened with a declaration of intent "to investigate the fundamental laws of those operations of the mind by which reasoning is performed"; while in an essay destined for *PL* "by the term Logic in its primary and most general sense we understand the Philosophy of the Laws of Thought as expressed" (page 126). However, the means of *expressing* thoughts was to be a major issue throughout his logical career.

§8 Between books, 1848–1854

Boole sent out copies of *MAL* to a few friends and colleagues. One of the first to respond was fellow algebraist Cayley, who had already been in touch over developing Boole's idea of an invariant, thanked him in December 1847 for the copy, noting the lack of a cancellation law (7.2) and wondering whether "Has $\frac{1}{2}x$ any meaning" (page 191). In the ensuing exchange Boole stressed his important notion of (non)-interpretability, making analogies between

$$\frac{x}{2} + \frac{x}{2} = x \text{ and } \sqrt{-1} \times \sqrt{-1} = -1 \quad (8.1)$$

where the combination of two uninterpretables yield an interpretable quantity; Cayley was not impressed.

In the preface to *LT* Boole was to state that he had written *MAL* quickly and "within a few weeks after its idea had been conceived"; in a paper of 1851 he had explicitly regretted its publication (1851a, 251). At some time he annotated two copies with additions and comments (Boole 1952a, 119-124; Smith 1983a), and he began to develop his theory in manuscripts. In 1848 he wrote one on "The nature of logic" which launches our Part A as Chapter I. Basing his task on the assumption that "The mind is not a passive recipient of the impressions of external things but it has the power of modifying its

own conceptions according to the laws of its own nature” (page 3), he expounded upon these laws in more detail than in *MAL*, though largely within the same framework (including on page 10 the same very universal Universe). He verbalised his index law by the case “good good men = good men” (page 7), which he repeated in *LT*, 32 and 43.

Around this time Boole also made his first attempt to write an “Elementary treatise” on his logic; it comprises Chapter II here. He stayed around syllogistic logic, and did not use his index law. However, instead of stressing its forms, he asserted instead that “in general we reason by signs. Words are the signs most usually employed for this purpose” (page 14). These words mark a conversion to the sign tradition in two respects: that signs are primary, and that “language affords the *signs* by which these operations of the mind are expressed and communicated”, as Whately put it (1826a, 55). It marked a significant change of emphasis from Boole’s neutral remark in *MAL* that “The theory of Logic is thus intimately connected with that of Language” (p. 5, typically Endowed with Victorian Capital Letters). Boole’s adoption of this tradition was permanent, although he gave the mind a more active role than was usual among its adherents.

In consequence of Boole’s new position, elective symbols were demoted in favour of classes; but in their turn classes were only one component, albeit a major one, in the supervening notion of language regarded as a “system of signs” (page 11). In a paper on “the calculus of logic” published in May 1848 in *The Cambridge and Dublin mathematical journal* (as the journal had been renamed in 1846), he reviewed the methods of *MAL* and marked this change by giving a greater presence to classes via this enigmatic identity (or equality?) (1848a, 126):

$$“x1 \text{ or } x = \text{the class } X” \quad (8.2)$$

Boole always restricted his use of language to nouns, adjectives and prepositions (page 14); nouns specified classes within the Universe, such as “men” within “humans”; adjectives similarly determined sub-classes, like “good men” within “men”. Prepositions expressed the connectives: “except” (–), exclusive “or” (+) and “and” (·). He could have continued this pattern by adding adverbs to his linguistic repertoire, forming, say, sub-sub-classes of “(not) very good men” from the class of “good men”. He seems not to have done so, however; maybe he realised that not all adverbs obey the index law: “very very good men” $\neq$ “very good men”.

So one must be wary of modernising Boole’s theories too much. The dash of denoting theory on page 16 does not make him a budding Russell, and he never envisioned propositional functions, or quantification in the sense in which we now understand the term. Similarly, his theory of classes was always the traditional one of part and whole; he never envisioned the set theory of Georg Cantor (1845–1918), which was come into mathematics only

in the 1870s, and not into logic until a decade after that, with the very different mathematical logic of Giuseppe Peano (1858-1932) (§14).

Another innovation in Boole's "Elementary treatise" was his distinction of "primary" from "secondary" propositions, the latter being "propositions which express a judgement with respect to other propositions" (page 29). The ensuing analysis was to develop in *LT* as a new theory of hypothetical propositions to replace (7.4) (§9).

Chapters I and II contrast in an important way. The first one treated logic as a science by basing it upon signs; but its successor dealt with "Logic as an Art" in "exhibit[ing] the most useful general forms in which valid argument may be expressed (page 13). The distinction was often stressed at this time; in particular, Whately started his *Elements of logic* with it. Boole was primarily concerned in his manuscripts with its scientific status.

Boole treated much of the subject matter of these two Chapters also in a collection of short essays or a suite of paragraphs on various themes, each given a title. Thus some essays, though maybe not all of them, date from this time. He wrote them in a notebook and removed the folios at a later stage. Chapter III contains from them selected "Extracts", as they shall be cited below.

One extract shows Boole relating signs to words in a language containing not only nouns, adjectives and prepositions but also verbs (page 48). Another one marks the début of his concern with the role of integers in his logic (Panteki 1992a, ch. 8); maybe Cayley's letter had initiated it. Repetition of x to produce $x^2, x^3, \dots$ leads to integers, similar to their presence in powers of differentiation in the other index law (6.3)₁; so do the expressions $2x, 3x, \dots$. He defended them on grounds of generality, both as constants and as symbols for operations, since raising powers is an operation in mathematics (page 44). He also examined laws of division, and thought about properties of inverses (page 45). He also pointed to differences, especially that the logical Universe 1 satisfied laws not possessed by the arithmetical 1 (page 45); the cause lies, of course, in the different index laws (6.3).

These notes also relate to an essay written in two notebooks which Rhees published in his edition (Boole 1952a, 141-166) and which we identify as "Sketch". In it Boole initiated his interest in probability theory; we have not included any other such manuscripts (there are in fact few of them), but a letter of 1849 to the astronomer and mathematician Sir John Lubbock (1803-1865) records the birth of his concern (pages 197-200). Thus he may have started the notes in or around that year; and continued the file for a period; the last extract planned out a book in four chapters to succeed *MAL* which somewhat resembles the "Elementary treatise" but also included the primary and secondary propositions (page 46).

A connected novelty in the notes is the (surprising) arrival of time in Boole's logical scene, due to "the repetition of an operation as in successive

Figure 7:
The house in Grenville Place,
Cork where Boole wrote *LT*
(Photograph I. Grattan-Guinness)

periods of duration” (page 44). In “Sketch” he came to think of his elective “symbols x, y, z , as representing the *times* in which the elementary propositions to which they refer are true”; thus x represented the proposition “it rains” by the symbolic proposition “ $x = 1$ ” (1952a, 146). One recalls W. R. Hamilton’s use of time in his definition of irrational numbers in 1837a; but their lack of contact (§2) suggests lack of influence. This theory hardly seems to be a philosophical improvement on the theory of hypothetical propositions in *MAL* (Hailperin 1984a); nevertheless he was to use it in *LT*, to which we now turn.

§9 Logic in *The laws of thought*

Boole wrote this book mostly at his residence in 5, Grenville Place, Cork (Figure 7). He may have finished it by 1852; it was typeset in Dublin during the next year, and published by MacMillan in Cambridge and Walton and Maberly in London early in 1854 (Neil 1865a, 165-166).¹¹ Boole and a friend

bore the expense (Boole, M. 1931a, 27);¹² it is unlikely that they recouped the costs, for it sold badly, much less well than *MAL*.¹³ Yet it exhibited a major extension of his logic. He seems not to have recognised the full consequences of his own ideas when writing *MAL*, in that syllogistic logic provided the main guide there; by contrast, it was relegated in *LT* to the last of the 15 chapters on logic.

The preceding 14 chapters were taken up with a much more general and detailed account of Boole's logic, both philosophically and mathematically. Although his psychologism (as specified in §7) was still fundamental, with signs as the key notion in his *Logic as a Science*, language was very prominent: for example, a major task was "*To deduce a consideration of those operations of the mind which are implied in the strict use of language as an instrument of reasoning*" (*LT*, 42).

Among these operations, Boole reduced the index law from (6.3)₂ to

$$x^2 = x \quad \text{with} \quad x(1-x) = 0 \quad (9.1)$$

as a principal consequence (pp. 32, 49). The reduction from the n -th to the second power was first stated in "Sketch" without discussion (1952a, 141, 149); he now argued that any higher power of x would lead after factorisation to the uninterpretable term $(1+x)$, which could therefore be discarded (p. 50n). The symmetry of (9.1)₂ as a function of x and $(1-x)$ led him to name it "the law of duality" (p. 51), and he deployed this important algebraic property in several places later on in the book. One of them was in his new theory of primary and secondary propositions, explained in ch. 4 and applied in chs. 11-12, with "Time" as a major concept. It also underlay his claim that "we can employ the symbol y to represent either 'All Y s' or 'All not- Y s', since the interpretation of the symbol is purely conventional" (p. 232) and allowed the absence of a symbol for "not".

The balance away from elective symbols towards classes continued strongly, especially after a proposal to read 0 as 0 y and 1 as 1 y (p. 47). Boole had already defined the Universe more carefully than hitherto as the "assumed or expressed limits within which the subjects of its [the mind's] operation are confined" in a given "discourse" (p. 42), a move heralded in the "Extracts" (page 44).

A major innovation, heralded in "Sketch" (1952a, 151-152), were the elimination theorems; for one and two variables respectively,

$$"f(1)f(0) = 0" \quad \text{and} \quad "f(1,0)f(1,1)f(0,1)f(0,0) = 0" \quad (9.2)$$

(pp. 101, 103). Boole's proof of the first one was rather ponderously algebraic; Harley 1871a was to show that it rested upon the theory of the roots of equations adapted to variables which took only two values.

The expansion theorems (7.5-6) were given a much more important place, and Boole now saw the role of all these theorems as not only solving (sets of) logical equations but also laying down conditions for any solution to

occur. For example with three classes, which class(es) z , if any, satisfies given relationships with given classes x and y , and under what conditions? Boole's method rested on taking all possible basic terms xy , $x(1-y)$, ... and using the appropriate version of (9.2) to calculate each of their coefficients c . If $c = 1$, then xy was part of z ; if $c = 0$, then it was absent from z ; if $c = \frac{0}{0}$, then any part " vxy " of xy would be part of z , which therefore could not be specified uniquely; and if some other value arose for c (often $\frac{1}{0}$; or 2, or 576.5 for that matter), then not only was xy no part of z but the condition $xy = 0$ was required for any classes z to be found at all. However, exceptional solutions could still escape the clutches of even this version of the method, as with (7.3). Hooley 1966a gives a method of obtaining all solutions of a linear equation in two-valued variables satisfying (9.2); Boole does not seem to have spotted it.

Chs. 16-21 of *LT* were concerned with probability theory, which we saw born in 1849 in §8; his main innovation was to interpret compound events as Boolean combinations of simple ones (and thereby increase the 'model-theoretic' character of his algebra). He also considered probabilistic inference, and in *estimating* probability values of logical consequences he manipulated inequalities in ways which anticipate linear programming (Hailperin 1986a). After *LT* he produced important papers in this area; unfortunately none of his contributions made a major impact.

The last chapter, "On the nature of science, and the constitution of the intellect", shows Boole's psychologism well to the fore: "what is remarkable in connexion with the processes of the intellect is the disposition, and the corresponding ability, to ascend from the imperfect representations of sense and the diversities of individual experience, to the perception of general, and it may be of immutable truths" (p. 407). Mathematics was at the heart of this; and psychology was influenced because the "mathematical laws of reasoning are properly speaking, the laws of *right* reasoning only" (p. 408). Among those laws, index and duality (9.1) were prime, with a long pedigree in ancient Greek philosophy (pp. 411-416); they also deployed "a scientific connexion between the conceptions of unity in Number, and the universe in Logic" (p. 411). Boole now tried to develop the philosophy of his logic within and around such considerations.

§10 Logic after *The laws of thought*

We treat now the manuscripts presented in Part B and the pertinent items from Part D, all written seemingly after *LT* but not destined for the third book, PL (§11). They are to be taken together with one which Rhees included in his edition under his title "Logic and Reasoning" (Boole 1952a, 211-229).

Science dominated Art in most of the manuscripts in this Part—explicitly so in the opening note of a collection forming Chapter VI (page 105). Boole continued there with a summary of a philosophical analysis of logic drawn largely from the sign tradition (he mentioned Locke on page 106); Whately had summarised it as a prologue to his account of syllogistic logic (1826a, 54-56). Boole presented it in more detail in a long essay of 1856 on “the foundations of the mathematical theory of logic”, presented here as Chapter V.¹⁴ In his version of the analysis he paid rather better attention than usual to the difference between a mental act and its product. The act of Conception by the mind produced a Concept, such as “men” (or the class of them); then the act of Judgement of the co-presence of Concepts produced a Proposition, such as “this is a wise man”; finally, the act of Reasoning produced a Conclusion, perhaps “wise men are ...”, as delivered by his algebra of logic (pages 88-95).

Throughout this essay, and indeed in all manuscripts assigned to this and the next Parts, Boole remained in his linguistic territory of nouns, adjectives and prepositions. To characterise mental actions he set aside elective symbols (and also time, one may be relieved to note) and envisioned logic as “conversant with things only as they fall under the general notion of Class” (page 67). Two pages later he even associated this commitment with the old and important notion of “second intentions” (or second-order predicates), which in his time still was ignored; he went back for it to an old source, Suarez 1605a. He also considered the intensional and extensional conceptions of classes (pages 74-76).

Boole rehearsed his basic “Laws of conception” in class terms, imposing his restrictions on addition and subtraction without discussion (pages 77 and 78 respectively). At a later point (page 92) he claimed a *proof* of the index law from the argument

$$(x+y)^2 = x+y; \quad \therefore xy+yx = 0; \quad \therefore xy = 0 \text{ as solution.} \quad (10.1)$$

However, as in (7.3), our logician has not sorted out his necessary from his sufficient conditions. He also deployed and discussed his expansion theorems and their enigmatic coefficients (pages 98-100, 187-188).

Alongwith his new allegiance to the sign tradition, Boole recognised the importance of syllogistic logic. In particular, in the essay of 1856 (page 72) he paraphrased from some edition of Whately that “since Logic is wholly concerned in the use of language, it follows that a Syllogism (which is an argument stated in a regular logical form) must be an ‘argument so expressed, that the conclusiveness of it is manifest from the mere form of the expression i.e. without considering the meaning of the terms’” (1826a, 88).¹⁵

In all these manuscripts Boole's type of psychologism remained in place. He developed one aspect in an undated note contrasting belief with understanding (Chapter XII), and planned out a two-page piece on the place

of freedom within his stern Science of Logic and the laws that the mind should obey (Chapter XIII).

§11 Plans and texts for "The philosophy of logic"

Despite the poor sales of *LT*, MacMillan's seem to have been ready to publish a sequel on "The Philosophy of Logic", and to have stated it to be "nearly ready" in 1857 (Neil 1865a, 172). But "the announcement was premature", Boole told De Morgan in March 1859 (Smith 1982a, 77),

I have written at different times as much as would make two or three books but when returning to a subject I can seldom make much use of old materials. They have lost their freshness & I can only begin again *ab novo*. And that is what I am doing — but — with a modest plan before me, having certain things to say & only desiring to say them. I am not going to set aside anything in the Laws of Thought — but only to interpret now within the province of pure Logic what is done there. When this is done I shall quit for ever.

But by then Boole must have been deep into his textbooks; in the remaining five years of his life, his other publications were several papers in the same area, and his final months were devoted to a second edition of the book on differential equations. It seems that he had stopped working on PL, perhaps in the early 1860s; he implied as much to De Morgan in a letter of November 1862 (Smith 1982a, 102). On the other hand he had definitely started; his wife had even helped by transcribing some manuscripts in the preparation for the press. There is at least a preface and a table of contents for a four-chapter work (Chapters VIII and IX); we place here also notes that may have been intended for a revised preface, and a long manuscript which looks like an attempt at the first two chapters (Chapters X and XI). The latter manuscript contains comments on De Morgan which suggests that he may have read De Morgan's paper 1860a; if so, it is among his latest writings on logic. No other manuscript seems definitely to belong to PL.

In his preface Boole stressed again that Logic as a Science was his concern (page 119). In the two drafted chapters he asserted that "the [C?]onception of class really occupies a position of priority in the intellectual order", but that its "narrower Logic of Class" would be developed "with constant reference to that higher Logic [...] as expressed by signs" (page 129); their laws "are a visible expression of the formal laws of thought" such as "the conception of any collection of things as a *whole*" (page 131). Further, in more detail than anywhere else he outlined and then criticised the traditional "Methods of Logic" (page 132) centered on syllogistic logic, both traditionally Aristotelian and as practised in his time by Sir William

Hamilton and by De Morgan (pages 133-141). His main criticisms of syllogistic logic were the emphasis on forms instead of signs, and words to express them, and its limited scope: he gave as an example its inability to state that equilateral triangles were identical with equiangular ones (page 136, perhaps to reinforce Whately's remarks on this example in *1826a*, 42).

De Morgan's contributions to logic are summarised in §13; a word on Hamilton's would be appropriate here. As was noted in §5, he popularised quantification of the predicate, and is often falsely credited as its inventor. In fact his contributions to logic and philosophy were slight, but he helped to raise interest in syllogistic logic, to popularise commonsense philosophy, and to bring German philosophy into Britain. In Boole's terms, he also saw Logic primarily as a Science, and took as basic the laws of identity, contradiction and excluded middle. Boole also agreed with him in construing Logic as an "unexclusive reflex of thought" (page 140), that is, an all-encompassing department of knowledge; he implied that he saw his own logic as more effective for the task. This all-embracing view of logic as Logic is notably different from modest modern approaches, where some logic among the hundreds now available is examined for its local (meta-)properties.

But Boole was quite at odds (at *MAL*, 11-12) with Hamilton's detestation of mathematics (Olson *1975a*, esp. pp. 62-71).¹⁶ Indeed, his foundations of Logic had to differ from those of all his contemporaries because his logic was applied mathematics, and so needed a mathematical basis. This survey of the selected manuscripts ends with his handling of that aspect of his philosophy.

§12 Where was Boole's philosophy of mathematics?

As algebra is an instrument of thought, applicable to our reasoning upon number and quantity, as its validity for this purpose depends upon the existence of laws such as [distributivity and commutativity ...], so is language an instrument of thought ...

Boole *1848d*

Boole's most extended manuscript on the philosophy of mathematics is rendered here as Chapter XV. Undated, it seems to be part of a book (though not PL), for it contains a "ch. 2" on geometry. He dwelt mainly upon Euclidean geometry, and suggested that our understanding of the different levels of generality of the assumptions in Euclid's *Elements* was not held by its author (page 177). His claim that "it becomes possible to present the entire procedure of the Science [of Geometry] in the forms of consecutive Syllogisms" (page 169) fitted poorly with his allegiance to the sign tradition. It also suggests that he had not read De Morgan's pioneering and far more sophisticated examination of the logic of Euclid's geometry, although it had

been reprinted in De Morgan's book *Formal logic* (1847a, ch. 1), which was published on the same day as *MAL* and of which he had received a copy. It may have been stimulated by a long discussion earlier in the manuscript of the role of definitions in mathematics; he saw them as "not description but distinction", between those things satisfying a definition and those not (page 172) — like the law of excluded middle, perhaps?

One issue in the relationship between logic and mathematics which Boole seems not to have considered is the danger of a consistency loop. Suppose that one wishes to found (a) logic *L* on a mathematical theory *M*. To be worth using, *M* should be consistent; but that is already a logical notion, no? Today we would instinctively switch into metalogic to handle this question, but that distinction was not properly appreciated until the 1920s. Curiously, in the course of analysing the general linear form (7.7) in *MAL*, Boole had brought up the possibility of inconsistency in the case of " $1 = 0$, which would indicate the non-existence of the logical Universe" (p. 65); but he did not pick up the point again. However, it lies at the heart of his task.

This was not the only lacuna in Boole's philosophy of mathematical theories. All of his points were necessary but not sufficient for the needs of his algebra of logic. Near the end of *LT* he had asserted, quite legitimately within his philosophy, "The truth that the ultimate laws of thought are mathematical in their form" (p. 407), and in his manuscripts he gave examples; for example from a fragment, that the expansion theorems came only from the basic laws, "not from a consideration of the possible relations of things to each other" (page 187). In another fragment "on reducing the Science of Logic in its technical development under the dominion of mathematical forms" (page 188), he noted the expanding realm of "the Science of Mathematics", which should be "no longer defined in its essential character by the nature of its subject matter but by the forms and method of its procedure" (page 189).

However, Boole was unable to find a philosophy of mathematics rich enough to ground his logic; and it may have been realising this, at least intuitively, which led him to stop working on *PL*. He needed something like a universal calculus of symbols within which both mathematics and logic could be sited (Laita 1977a); but he never found it, and in any case the consistency loop might not be broken. He might have found something in the semiotic tradition in France at the turn of the centuries, in which the Abbé Condillac (1714-1780) was father-figure to a group of *idéologues* who emphasised the place of signs on knowledge and thought,¹⁷ and also regarded (common) algebra as the language *par excellence* (Panteki 1992a, ch. 1). However, like his contemporaries, Boole did not pick up this tradition, although its own parentage goes back to Locke (Auroux 1981a).

During the late 1850s Boole was spending the majority of his research time on preparing his textbooks on the differential and difference equations; but its subject matter was bound closely to his philosophy of logic by the

common factors of signs, operations and language. In the early "Extracts" he gave an example from a method of evaluating a type of multiple definite integrals due to J. P. G. Lejeune-Dirichlet (1805-1859), to which he was alerted by Cayley (Boole 1848b, 140). In this method the integrand was multiplied "by a factor of which the value is equal to unity in the range which the integrations must cover, and which vanishes outside of this range" (Dirichlet 1839a, 375); Boole saw this assignment of 1 or 0 as a "good example of the real nature of an elective symbol" (page 45).

A more substantial example is singular solutions of differential equations, which Boole was recalled as teaching at Cork "not like a professor writing a demonstration on a blackboard, but like an artist painting from a vision" (Boole M. 1931a, 76). Why? As he wrote in his textbook on differential equations (1859a, 140), for "the two marks, positive and negative, by the union of which [they] are characterised": solving the equation but lying outside its general solution — a perfect image of $x + (1-x) = 1$. During his last summer he wrote to his wife that his work in the Library of the Royal Society had brought the theory "in a state of unity" (Boole, M. 1931a, 444), and he studied not only recent work but also the history of the topic (Boole 1865a, 9-37).

Elsewhere in that textbook occurred an even closer link (1859a, 398-399). The example in hand was $f(\frac{d}{dx})$, but this passage could have been in PL.¹⁸

In any system in which thought is expressed by symbols, the laws of combination of the symbols are determined from the study of the corresponding operations in thought. But it may be that the latter are subject to *conditions of possibility* as well as to laws *when possible*. And thus it may be that two systems of symbols, differing in interpretation, may agree as to their formal laws whenever they both express operations possible in thought, while at the same time there may exist combinations which really represent thought in the one, but do not in the other. [...]

Now all special instances point to the conclusion that this is permissible, and seem to indicate, as a general principle, that the mere processes of symbolical reasoning are independent of the conditions of their interpretation. In the few instances we may have occasion to employ, verification will be easy. We take occasion to notice that whatever view may be taken of this principle, whether it be contemplated as belonging to the realm of *a priori* truth, or whether it be regarded as a generalization from experience, it would be an error to regard it as in any peculiar sense a mathematical principle. It claims a place among the *general* relations of Thought and Language.

§13 Outside Boole's universe: Leibniz, Peacock, Babbage and De Morgan

From lacunae to omissions: these four figures are often associated with Boole in some way; yet they appeared but little in his publications of manuscripts. The reasons provide additional perspectives on his work.

Firstly, G. W. Leibniz (1646-1716) arises not only for his dream of a universal language including logic but especially for a passage (transcribed in the textual notes on page 219) in a manuscript *1681a* in which he expressed a law similar to the index law (9.1)₁: in those notations, it reads $xy = x$, with x and y as terms and the product read as conjunction. Moreover, it had been published, for the first time, in J. E. Erdmann's edition (Leibniz *1840a*) of philosophical writings; so it was available to Boole (Peckhaus *1996a*). However, it did not exercise influence upon him. Apart from the possibility that he had already envisioned such laws from considerations of monism (§2), he had not seen the edition, and he was informed of the passage by his friend Robert L. Ellis (1817-1859) only in 1855, a year after *LT* appeared (Harley *1867b*, 5; compare Boole, M. *1931a*, 1132). Boole's first reactions are reproduced here among the fragments (page 185 and the self-confessional page 188).

In any case, Leibniz had not developed his remark into any *theory*, so that the anticipation is not substantial. In any case, which theory? When Peano began to develop his mathematical logic in the 1880s, he also cited Leibniz as father-figure ...

Previously Boole had not mentioned Leibniz much; a few places in passing (for example, pages 133 and 179 here), and on the law (7.1)₁ of contradiction from a Leibniz publication (*LT*, 240: in Chapter XIV he noted this law in Aristotle). Despite Erdmann, the logical and philosophical writings of Leibniz were far less well known then than they are to us, in the wake of Couturat *1903a* and later research. Indeed, Leibniz was still regarded in Britain as the stealer of Newton's calculus, although De Morgan had recently started to undermine that story with proper historical research (Rice *1996a*).

Our next two figures were founders of the "Analytical Society" at Cambridge (1812-1813), who played an important role in the revival of English mathematics by adopting Continental methods. George Peacock (1791-1858) later wrote a large "Report on the Recent Progress and Actual State of Certain Branches of Analysis" (*1834a*), which historians often cite as a landmark text in setting the stage for the development of English algebras. His emphasis on symbolic methods and the (non-) interrelativity of a formula

in a given algebra doubtless reinforced awareness of the issues raised by the French mathematicians cited in §6. His “principle of the permanence of equivalent forms” asserted that if an equation $A = B$ held in an algebra for specific conditions on the components of the expressions A and B , then it held in general; and Boole’s analogies such as (6.1-2) certainly provided example. But *citation* of Peacock by the English is less common than may be expected; in particular, Boole seems not to have mentioned him at all. This silence is in fact somewhat surprising; for Peacock had emphasised the role of signs in algebra, and his line of thought shows some kinship with Locke (Durand 1990a), and so with the sign tradition.

The third figure is Peacock’s Analytical colleague Charles Babbage (1791-1871). He is sometimes mentioned with Boole; for his long concern with calculating machines, which ran on throughout Boole’s youth and adulthood, seems so akin to our use of Boolean algebra. However, the two men had no particular congress, and no manuscripts of one seems to dwell on the other. On Boole’s side, as we have seen, in both logic and for educational practise he thought that the mind was capable of original action, such as grasping general laws from particular cases (*LT*, 4; Boole, *M.* 1931a, 5-6), and so he would not have welcomed the association of his logic with the *repetitive* actions of computing; whether Babbagean mechanical or modern electrical. For his part, Babbage disliked Boole’s practise of allowing the algebra of differential operators to be interpreted as operations and as objects.

This contrast is stronger than it may seem; for in mathematics Boole and Babbage had an important common factor. In the 1810s young Babbage had worked on another new algebra encouraged by Lagrange’s foundation of the calculus (§6) — functional equations, in which functions rather than unknown constants were sought. For example, find the functions f (if there are any) for which

$$f(x+y) = f(x)f(y) \text{ for all real values of } x \text{ and } y. \quad (13.1)$$

The theory attending this algebra contained various features which are evident also in his work on computing; iterations, algorithms, and especially signs, whether physical, mathematical or mental (Grattan-Guinness 1992a).

The first systematic account of functional equations was written by our fourth figure in this section: De Morgan, in a long article 1836a for the *Encyclopaedia metropolitana*. It must have influenced his most original contribution to logic, in 1860, a syllogistic logic of two-place relations (Merrill 1990a), for the analogy between this logic and functional equations is strong; inverses, compounding, and so on. Boole used functional equations occasionally, such as when finding his general expansion theorems (7.5-6) (*MAL*, 60-63), but he never noticed this important aspect of logic. Nor did he seem to react to De Morgan’s paper, although by 1860 his main interest lay in differential equations.

Previously De Morgan had extended considerably the realm of syllogistic logic by applying various algebraic techniques, of both theory and signs (Panteki 1992a, ch. 6): the quantification of the predicate (§5) is an important example. (De Morgan 1966a is an edition of his main papers.) An algebraist by mathematical inclination, he and Boole were the most significant algebraic logicians of their time, and each man was influenced by a new algebra. But their two approaches were quite different; although they corresponded regularly (Smith 1982a), neither used the other's logic, and they communicated more than discussed (Corcoran 1986a). Thus Boole's comments on pages 138-141 noted in §11, his most extended statement on De Morgan's logic, were not sympathetic.

§14 Boole's immediate legacy

While always staying with his psychologistic view of logic, in the sense explained at the end of §7, Boole made various changes while seeking its foundations. The most significant one was away from syllogistic logic to the sign tradition; language gained ground along the way, and so did classes, at the cost of elective symbols. Time made an appearance, though seemingly for a short duration. The algebra received a very original (although not complete) development; its central place in the logic required a rich mathematical basis, which he was not able to find.

Boole's contemporaries knew little or nothing of his manuscript ruminations. After his death, his reputation rested largely on his textbooks in differential equations, which continued to be well used after his death and indeed are still in print. But his differential operator methods became generally eclipsed by other techniques from the 1870s, and his work on logic remained rather marginal (Panteki 1992a, ch. 9).

Boole's most important initial follower in logic was Stanley Jevons (1835-1872); they corresponded in 1863 and 1864, just before his death. The main topic was considerable disagreement over "+", using " $x+x$ " as a test case (Grattan-Guinness 1991a). Jevons had no time for Boole's restriction, which rendered the expression uninterpretable¹⁹; in his book *Pure logic* (1864), he wanted to allow that

$$x+x=x. \quad (14.1)$$

Shortly after Boole's death the chemist Benjamin Collins Brodie (1817-1880) tried, unsuccessfully, to develop a similar algebra for chemical combination (Brock 1967a). John Venn (1834-1923) was the sole close adherent in Britain, mainly in his textbook *Symbolic logic* (1881, 1894); to make the theory more easy to grasp he introduced diagrammatic representations of syllogisms — not the ones which are misnamed after him,

which are due already to Leonhard Euler, but a rather clumsy procedure in which all possible overlaps of convex shapes corresponding to the given predicates are shown and the non-empty ones in a given set of premises are examined.²⁰

Among other British semi-followers, Hugh MacColl (1837-1909) made an innovation that was to be so durable that Boole is sometimes credited with it: that "The calculus of equivalent statements", or the propositional calculus, could be construed as a Boolean algebra, with addition and multiplication read respectively as disjunction and conjunction, and equality as equivalence (MacColl 1877a). This proposal, and the differences between Boole and Jevons, were among the topics reviewed in a lengthy and influential survey 1892a of "The logical calculus" made in the philosophical journal *Mind* by the Cambridge logician W. E. Johnson (1858-1931). A few years earlier Mrs Sophie Bryant 1888a had swung against the trend in its pages, claiming that Boole's logic should be understood as an operational calculus and lamenting the preferred reading in terms of classes.

Outside Britain, Boole's logic gained some publicity during the 1870s. The Frenchman Louis Liard (1846-1917) wrote an article 1877a on it, and followed up with a book 1878a "On the contemporary English logicians" which became quite popular although logic was not in general a well received topic in France. G. B. Halsted (1853-1922) performed a similar service in the USA with a survey article 1878a (and also a piece 1878b in *Mind* on Jevons's modifications).

The most significant advance began also in the 1870s: the fusion of Boole's algebra of logic with De Morgan's logic of relations. This was effected by Halsted's compatriot C. S. Peirce (1839-1914), who also modified addition to (14.1), independently of Jevons. The Scottish mathematician Alexander MacFarlane (1851-1913) made a modest contribution of this kind in a short paper 1879a (and in a contemporary book 1879b he modified Boole's logic by distinguishing nouns from adjectives).

In Germany the German philosopher Hermann Ulrici (1806-1884) gave Boole's logic publicity, starting out from a long review of *LT* in 1855 (Ulrici 1855a). It also came to bear upon a massive enterprise of algebraic logic undertaken by Ernst Schröder (1841-1902); but he drew more directly on the algebra of Hermann Grassmann (1809-1877) and the logic of brother Robert Grassmann (1815-1901), and especially on Peirce. This work was taken up in turn. In Russia from the mid 1880s, principally with P. S. Poretskii (1846-1907) (Styazhkin 1969a, 216-253).

However, from the beginning of this century the whole algebraic tradition in logic began to be eclipsed by the development of mathematical logic with Peano and his school and then with the logicism of Russell and Whitehead. It drew on completely different principles, including Cantorian set theory, and furnished mathematical quantities such as real numbers and lengths as well as qualities such as true and false; indeed, logicism was logic

applied to mathematics instead of Boole's vice versa (Grattan-Guinness 1988a). The failure to produce an edition of Boole's manuscripts in the 1900s (§3) was a sign of those times in symbolic logic.

The psychological and religious aspects of Boole's logic largely disappeared with him, and they do not feature in his manuscripts. Perhaps he could not see how to build them in effectively — and since they came from his Dissenting heart, how to convey them honestly. But there seems no doubt that they always lay close to the centre of his logic. Boole's Boolean algebra was another new algebra in the time of their proliferation, but one set apart from the others not only by the index law $(6.3)_2$ but especially consequences such as (7.1) involving the all-important Universe 1. This remained with him from youth throughout his mature career, especially in the image of the unique Godhead and the ecumenism encapsulated by Frederick Denison Maurice. As he lay dying, in only his 50th year, he had Maurice's portrait set by his bedside (p. 48):

I said, "Are you still willing to leave the future entirely to God?" He nodded, and there came over him a smile of such wonderful peace that I would not speak again; and so he lay quiet until the end.

Part 2: Boole's Psychologism as a Reception Problem

Gérard Bornet

§15 Introduction

More than 125 years after Boole's death, understanding his work is accompanied by many difficulties of interpretation arising from the differing backgrounds of the reader on the one hand and the author on the other. The greatest obstacle to an adequate reception today is surely the psychologism that pervades all Boole's logical work (§§7, 9, 10, 14). In consequence we have sought to prevent an over-hasty dismissal of Boole's position by the reader

and to encourage a close look at what Boole is actually asking at his time rather than a simple change of subject.

In this second part we briefly discuss three aspects in which reception of Boole is hampered in connection with the word “psychologism”. First (§16) the epistemological framework in which the German mathematician and logician Gottlob Frege (1848-1925) poses the problem of psychologism. Then (§17) we show that Boole must be exonerated from Frege’s criticism. The second aspect deals with the question of art versus science. Here, the fact that in the course of time the centre of general interest shifted from logic as science to logic as art (§18) causes a reception problem. Doing Boole justice means considering his psychologism as part of logic as science (§19).

The problems touched on so far relate to the change in the understanding of logic in general and may crop up in connection with other thinkers of this era. But the third aspect is essentially Boole’s own peculiarity and addresses difficulties faced even by his contemporaries. In this connection today’s attitude is to see these features of Boole’s system as quirky and therefore to be neglected. It will first be pointed out (§20) that Boole does not use the psychological terms of ordinary language in their full verbal meanings (compare also page 76). Then (§21) the existence of a whole group of signs central to Boole’s system must be emphasised; signs which can have no significance at all in the psychologistic sense because they are “uninterpretable” in accordance with definition.

§16 Frege’s psychologism criticism

Gottlob Frege, Professor of Mathematics at the Jena University 1879–1918, is regarded as the founder of modern logic and philosophy of language. His work influenced thinkers like Ludwig Wittgenstein (1889–1951) and Rudolf Carnap (1891–1970) so deeply that he is the so-called father of today’s analytical philosophy. A characteristic of his teaching included a strict anti-psychologism (of which more later) and so an avoidance of anything that awoke the merest suspicion of psychologism became a trait of his following. Thus among the ranks of the most likely candidates for interest in Boole as a philosopher, the use of psychological terms in mathematics and logic were no longer seen as *de rigueur*.

However, the sweeping verdict is at times applied to variations not actually included in Frege’s criticism, among them, as we shall show, those of Boole as the author of *Laws of Thought*. For example Bertrand Russell ludicrously suggested that Boole had given *LT* the wrong title: “He [Boole] was ... mistaken [!] in supposing that he was dealing with the laws of thought: the question how people actually think was quite irrelevant to him ...” (1901a, 366). Many later authors who, like Russell, mentioned Boole only

in passing, had, with him, never read Boole's major work,²¹ having been put off because the book's title seemed to signal contents irrelevant to logic, namely the question of "how people *actually* think".²² But it was specifically to avoid misunderstanding in this respect that it was declared, in §7 above, that Boole wanted to give "a *normative* account of the *correct* use of thinking" (our italics).

The false reading of the title testifies to the triumphal march of experimental psychology, which barely existed in Boole's time. The first institute in this field was established for Wilhelm Wundt (1832–1920) in Leipzig in 1875. Up to that time psychology and philosophy had in common that they typically tackled problems theoretically. And as late as the early 20th century, it was not unusual for one university chair to cover both disciplines. In his criticism during a period of radical change, Frege was basically attempting to claim a separate field of study for Logic. Thus he pressed in 1918–19a (implicitly already in 1884a) for an autonomous *third dominion*. In this way he could make logic independent of nature on the one hand and individual souls on the other, thereby freeing himself from the authority of Physics and Psychology. Interestingly, a similar move by Boole is found in a fragment of the Nachlass (MS B99.7):

The phenomenal study of things belongs to physics or to psychology, the inquiry into their absolute nature if indeed such an inquiry be possible is the business of metaphysics. Logic has other objects and is concerned with other relations – but as the relations with which it is concerned are universal (for all existing things can be contemplated under the notion of class or kind) Logic stands related to all other sciences.

As is well known, Frege (particularly in 1884a) put forward two arguments against a psychologistic foundation for mathematics: that it robbed mathematical reasoning of (a) its *strict truth* because it must be inductively gained, and (b) its *general validity* (in the sense of intersubjectivity) because it would in the end relate to subjective facts. His arguments are so fundamental that they disprove the possibility of a psychologistic foundation for any science – and in this sense they can be applied to Logic too. By insisting on strict truth Frege excludes the use of experimental methods in mathematics; and by stressing the point of general validity Frege makes it obvious that the advocate of psychologism is left with nothing he can objectively speak about.

Consequently only those varieties of psychologism fell under his verdict which (a) used inductive methods or (b) related to subjectivity, that is affections of individual human souls. In no way did Frege intimate that all dealings with thought necessarily drew his criticism. On the contrary, in his *Begriffsschrift* he himself proposed a formula language for pure thought, offering the best counter-argument. It remains to be added that despite the

fact that Frege did indeed criticise Boole (§18), he did not reproach him for psychologism. Knowing how quarrelsome the Professor of Jena was, that equals exoneration.

§17 Boole's anti-inductivism and the problem of introspection

In fact neither of Frege's anti-psychologistic arguments applies to Boole though we can here only conclusively prove this for the first, the one from strict truth. For the second, the one from general validity, the reader is referred to page 68 or 1952a, 226, where Boole anchors the laws of logic in the intelligible field, which is per se intersubjective. A psychologistic reading of Boole in the line of the second argument is further also invalidated by the circumstance discussed in §21 that Boole admitted signs which he expressly described as uninterpretable – by which he meant that their meaning in the psychologistic sense was not accessible to the human mind.

Frege's first argument is quickly demolished for here Boole is unmistakable (*LT*,4; compare also page 32):

... the knowledge of the laws of the mind does not require as its basis any extensive collection of observations. The general truth is seen in the particular instance, and it is not confirmed by the repetition of instances. [...] a general truth in Logic ... is made manifest in all its generality by reflection upon a single instance of its application. And this is both an evidence that the particular principle or formula in question is founded upon some general law or laws of the mind, and an illustration of the doctrine that the perception of such general truths is not derived from an induction from many instances, but is involved in the clear apprehension of a single instance. In connexion with this truth is seen the not less important one that our knowledge of the laws upon which the science of the intellectual powers rests, whatever may be its extent or its deficiency, is not probable knowledge. For we not only see in the particular example the general truth, but we see it also as a certain truth,—a truth, our confidence in which will not continue to increase with increasing experience of its practical verifications.

Naming no names (cf §7) this was certainly aimed at Mill, just the person Frege (1884a) explicitly targeted with his first argument. In this instance Boole the seeker of the laws of thought is at one with the creator of a language for pure thought.

However, there are also differences in their positions. Among other things, Frege designed his *Begriffsschrift* for the purpose of being able to do

away with intuition in proof. He succeeded in this by making the unbroken chain of evidence demonstrable by signs and rigorous detailed proofs. Boole on the other hand, in the intellectual field too, speaks of “observation” and “perception” as in the quotation above (*LT*, 4). Today’s reader will, however, not be content with the then-current usage of terminology and will require a more exact picture of the mental processes described. Plain from his presentation is in fact only that a “usual” observation cannot be meant, as otherwise it would remain incomprehensible why a repeated examination should not yield a sharper picture.

On the problem of introspection as here addressed, little information is available in Boole. Most interesting is section 9 of the introductory part of chapter iv (pages 52-53) where “facts of external observation” are opposed to “facts of consciousness”. In this connection an “appeal to consciousness” or a reference to “the direct testimony of consciousness” appears as a matter of course. But Fragment 1 on page 179 reveals that Boole was quite aware of the special importance of the ability of introspection. If we find him here far away from a satisfactory answer, we must honestly ask ourselves whether one exists at all. For Boole, more closely considered, touches on one of the “eternal” questions of philosophy. Our commentary in §18 points out that progress since Frege, broadly speaking and confined to the matter in hand, lies not in better philosophical answers but simply in the avoidance of the relevant questions.

§18 Art versus Science

Skill in the processes of art or science applied is one thing. A thorough comprehension of the grounds upon which those processes rest is another. (Boole, page 124)

The *art* of logic consists in *using* a given symbol system in one sphere. To this, skills in symbol manipulation must also be added, for the greater the skill of working in symbols, the easier is the employment of them. Against that, *science*, in its usual meaning, delivers *reasons* and says *why* something is as it is (compare page 13). That is in every way a philosophical problem, and in Boole’s time “philosophy” and “science” were nearer together than they are today. Natural science instruments were wont to be described as “philosophical instruments”, and Boole himself speaks repeatedly of a “philosophical language”, meaning thereby a “scientific language” (e.g. *MAL*, 5). In the epistemic position of Naturalism this close relationship between philosophy and (natural) science is still alive today.

In the contention between art and science, two problems for the reception of Boole present themselves. In the first place, since Frege logic as art has developed, bringing with it a natural tendency for later recipients to regard earlier logicians as “artists” and not as “scientists”. And secondly this

has been strengthened by Boole's own attempt to dissociate himself from metaphysics (see quotation from MS B99 in §16); this despite the fact that the borders between metaphysics and fundamental science come down to the simple expression of a value judgement. This section deals mainly with the first point, while the second forms the kernel of the next one. However, one remark should be made in advance.

When Boole emphasised that he was not dealing in "the research of causes" (already in *MAL*, 11ff), it was mainly with the purpose of avoiding metaphysical discussion. In this sense he quite openly declares in *LT*, 11 that (in his book) he would not provide an answer to the question *for what reason* "the ultimate laws of Logic are mathematical in their form". And he continues in a distinctly audible undertone: "Such knowledge is, indeed, unnecessary for the ends of science, which properly concerns itself with what is, and seeks not for grounds of preference or reasons of appointment." Taken literally and isolated, this would show that Boole was not a scientist in the sense first indicated above. But that he is in fact such, is, for example, clear from page 141, where we find him searching for *ultimate* laws of thought (emphasis added). For him the difficulty in a "science of logic" lay not in the ascertainment of general truths of logic – these he recognised at a glance – but in the definition of those which are "primary and fundamental" (*LT*, 4f).²³

In the public consciousness, however, such statements remained by the way: Boole was above all known as the author of *MAL* (§2), that is to say as an "artist". This fact led to the philosopher Edmund Husserl (1859–1938) being able without quibble to offer him as an example of a person who could be "a splendid logical technician" and at the same time "a very middling philosopher of logic" (1891a, 9). This assessment has something tragic about it for Husserl was another such man, who, at the turn of the century, came down most heavily on the side of a logic as science. He would have been highly delighted by Boole's differentiation between a *noeton* (i.e. an intelligible) and an *aistheton* (i.e. a sensible) (page 68 or 1952a, 226) but the writings in which Boole sought to bring this distinction to fruition had not then been published.

So we see how Boole, in the eyes of later recipients, fell between two stools. By not getting to the grips with metaphysical general truths of logic, he was dismissed as a "middling philosopher" by those interested in such truths. And since logic as art had bounded forward with Frege, he soon seemed a "middling technician" too (in contrast to Husserl). And so Frege discerned, before all else, in Boole "lack of art"²⁴, as the following quotation (1880–81a, 12) testifies to:

If I understand him aright, Boole wanted to construct a technique for resolving logical problems systematically, similar to the technique of elimination and working out the unknown that algebra teaches. To this end, he represents judgements in the form of equations that he constructs out of letters and arithmetical signs such as +, 0 and 1. ...

In the main these means fulfil their purpose, at least as far as the range of problems that Boole has in mind are concerned. But one may think of logical problems lying outside this range.

Frege further found fault in that Boole's system for mathematical reasoning was "completely unsuited" if only for the fact that "[Boolean logic] employs the signs +, 0, 1 in a sense which diverges from the arithmetical one. It would lead to great inconvenience if the same signs were to occur in one formula with different meanings." (1880a, 13). As right as Frege is here, his objections refer to Boole's *art* of logic alone and not to his *science*, which is not mentioned at all.

§19 Psychology as science

So much is plain: psychological considerations get logic as art no further. We shall see in §21 that Boole of all people was aware of this. But in the realm of logic as science the role of psychology is different. Here it provides not skills but the necessary answer to the question of the subject area covered by the science being referred to. Insofar as this question is thought admissible, it can, even today, only be satisfied with a psychological response.²⁵ For as Boole correctly remarked the term "logic" comes from the Greek word λόγος, where it "signifies not only the inward thought but also its outward form or manifestation" (page 126).

However, the relationship between art and science has changed beyond recognition. Although it is today possible as a logician to devote oneself to art for art's sake and, say, concentrate exclusively on the syntactic properties of a calculus, in the first half of the 19th century such an attitude was absolutely unthinkable. Joan L. Richards explains why English mathematicians did not develop an abstract algebra as understood today despite possessing all the elements for it. Because such an algebra was not a goal worthy of endeavour (1980a, 345, emphasis added):

The issue which separates early English algebraic development from modern algebra is ultimately grounded in a fundamental difference between 19th- and 20th-century views of truth. For an English mathematician in the early 19th century to devote himself to developing an abstract algebraic system would have required a major change not only in the interpretation of the nature of mathematics, but more generally in the view of human knowledge. ... In the 19th-century English view, mathematical investigation involved the scientific study of an *external subject matter*, and not merely the logical development of theorems from given axioms and definitions. The truth of a mathematical investigation, the goal after which the mathematical researcher was striving, lay in the *subject*

matter his axioms and theorems *described*. The empty form of the mathematical structure had no truth or value of its own.

Without external subject matter, no science. On this and no other grounds, W. R. Hamilton (1853a), among others, felt obliged to develop algebra as "the science of pure time" (§8). It was not his interest in time that swayed him, but the status of algebra as science (also see Øhrstrøm 1985a). A proper assessment of Boolean psychologism must take this status into account as well.

Anyone wishing to take Boole's demand to be seen as a scientist seriously must also take note of a premise widespread (though not undisputed) at this time: the possibility of a science being strict. This pre-supposes the means of statements which are necessarily valid and which despite being based on experience cannot be refuted by this. Such statements cannot be inductively gained (as we observed already in §16 in connection with Frege). If such a supposition seems rather strange to us today, it must be remembered that only sixty years lie between the publication of Boole's *MAL* and Immanuel Kant's "Copernican Revolution" (1787a), in which the latter sought to rescue strict science from scepticism. And so Boole could confidently write in *LT*, 3:

It is unnecessary to enter here into any argument to prove that the operations of the mind are in a certain real sense subject to laws, and that a science of the mind is therefore *possible*. [...] Let the assumption be granted, that a science of the intellectual powers is possible, and let us for a moment consider how the knowledge of it is to be obtained.

From the acceptance of a possibility of a strict science of the intellectual powers springs more than at first meets the eye. For instance, such a science is inconceivable without a suitable means of expression. It was obvious that Boole, unlike us, would see these means in the language of mathematics, for had this not already made physics a "strict science"? Psychology can be practised as a strict science only if its fundamental laws have a certain simplicity in the manner of Boole's "Laws of Conception" (pages 76ff). Laws of experimental psychology are, in contrast, neither necessarily valid nor simply formulated.

In order to understand the role the mind plays in logic as science for Boole, he must be firmly placed in the "sign tradition" (§5). In this tradition the processing organ for signs is the mind, not the brain. It would never have occurred to Boole to found his logic on the "general laws of nervous actions" as, for example, Peirce did in 1880a. Such speculation did exist at Boole's time. For instance, Dr. Alfred Smee (1818–1877) proposed an "adaption of process of thought to algebraic formulæ" in 1851a which is "electro-biologically" grounded. Again, I. P. Hughlings (dates not known) could not resist enriching this introduction to the then-modern logic 1869a

with thoughts on nervous actions. So it is hardly surprising that Boole's psychologism was associated with such thought processes. Yet it struck out in a quite different direction in his philosophy. And thus we come to that trait in Boole's teaching that is most often overlooked or misunderstood.

§20 Boole's structuralism

The fact that Boole did not have the structural terms of modern mathematics at his disposal has already been noted (§19). There is, however, in *philosophy* a more general concept of structure whereby the expression "structure" describes the *relationship* between elements of a composite whole. In the 20th century, structures in linguistics, ethnology and psychology have been examined in this sense. Boole himself did not speak of structures and treated them first and foremost as not having an independent existence. He comes over as a precursor of structuralism, however, when he publicly declares (*LT* 39): "The object of science, properly so called, is the knowledge of laws and relations". Of the entities connected through these laws and relations he intentionally says nothing.²⁶ A fine example of Boole's "structuralism" can be found on page 157 (the inverted commas indicate a pre-form, not modern coinage), where the author, in a discourse on belief, is not concerned "to investigate the nature of that mental act which we designate belief, but to state some of its general relations."

If science is concerned only with the structure of its subject, an interesting and almost always overlooked consequence arises for the meaning of normal language terms: they are over-defined. Boole illustrates this with the example of controversy regarding cause and effect (*LT*, 39). One person may see nothing deeper in this than a regular succession in time whereas another might insist that there is an innate connection between the perceived phenomena. The two persons may endlessly quarrel over denotation, but even if the one accepted the words at face value and the other sought deeper significance in the nature of causality, they would be agreed on the fact that one of the two phenomena involved followed the other in time. And with this they recognise, according to Boole, "a common element of scientific truth, which is independent of their particular views of the nature of causation." If Boole were to speak of causality – to stay with this example – he would mean with this term only the (uncontroversial) "common element of scientific truth".

This observation is as valid in the sphere of semiotics for the argument between conceptualists and nominalists (page 70f) as in the sphere of psychology, where Boole sets the position of the idealists against that of the sceptics. Here too, there are common threads – and only these interested him as a scientist (*LT*, 40):

Let it even be granted that the mind is but a succession of states of consciousness, a series of fleeting impressions uncaused from without or from within, emerging out of nothing, and returning into nothing again,—the last refinement of the sceptic intellect,—still, as laws of succession, or at least of a past succession, the results to which observation had led would remain true. They would require to be interpreted into a language from whose vocabulary all such terms as cause and effect, operation and subject, substance and attribute, had been banished; but they would still be valid as scientific truths.

Boole is clear that the terms used “in the science of the intellectual powers become, in expression at least, almost necessarily mixed up with the modes of thought and language, which betray a metaphysical origin”. (*LT*, 39). Nevertheless, he did not renounce employment of the suspect terms (*LT*, 41):

The course which it appears to me to be expedient, under these circumstances, to adopt, is to avail myself as far as possible of the language of common discourse, without regard to any theory of the nature and powers of the mind which it may be thought to embody. For instance, it is agreeable to common usage to say that we converse with each other by the communication of ideas, or conceptions, such communication being the office of words; and that with reference to any particular ideas or conceptions presented to it, the mind possesses certain powers or faculties by which the mental regard may be fixed upon some ideas, to the exclusion of others, or by which the given conceptions or ideas may, in various ways, be combined together. To those faculties or powers different names, as Attention, Simple Apprehension, Conception or Imagination, Abstraction, &c., have been given,—names which have not only furnished the titles of distinct divisions of the philosophy of the human mind, but passed into the common language of men. Whenever, then, occasion shall occur to use these terms, I shall do so without implying thereby that I accept the theory that the mind possesses such and such powers and faculties as distinct elements of its activity. Nor is it indeed necessary to inquire whether such powers of the understanding have a distinct existence or not. We may merge these different titles under the one generic name of *Operations* of the human mind, define these operations so far as is necessary for the purposes of this work, and then seek to express their ultimate laws.

So Boole argues that there is an ideal language, believing himself to have discovered it in the form of the then-current algebra. His psychologism, however closely it may be defined, cannot be made a case by simply pointing to the notorious woolliness of the psychological terms involved. For not only did he himself draw attention to this, he even sought clarity through “translation” into an “ideal language”.

§21 Uninterpretable symbols

This last aspect of Boole's psychologism provoked controversy from the very beginning. Soon after *MAL* was published Arthur Cayley raised queries (§8, Chapter XVII). Again in 1863, the subject reared its head in correspondence with Stanley Jevons (§14; Grattan-Guinness 1991a, 26f.) and later Rudolf Hermann Lotze in 1880a (quoted in Hesse 1952a, 76) made insinuations.

What is it all about? Boole applied algebra for the purpose of logical reasoning by forming the premises into algebraic equations, solving them and transcribing the result into a conclusion. That many found the intermediate steps logically uninterpretable did not only not worry him at all (compare pages 43 and 146-148), but indeed was a circumstance to be taken into account, otherwise it would be "manifest, that no such thing as a general method in Logic is possible". (*LT*, 67). At the same time, Boole conceded,

... that this apparent failure of correspondency between process and interpretation does not manifest itself in the *ordinary* applications of human reason. For no operations are there performed of which the meaning and the application are not seen; and to most minds it does not suffice that merely formal reasoning should connect their premises and their conclusions; but every step of the connecting train, every mediate result which is established in the course of demonstration, must be intelligible also. And without doubt, this is both an actual condition and an important safeguard, in the reasonings and discourses of common life.

The last sentence of the quotation betrays the source of a fundamental misunderstanding in the assessment of Boole's psychologism. For there is almost an acceptance that anyone who psychologically interprets premises and conclusions of logical reasoning grounds the *validity* of the final account by making the steps on the way intelligible. Yet for Boole, the validity of the *logical* ending did not depend on given psychological factors but on the laws of symbols (compare page 70).²⁷ That is why, when he later sought popularisation, he defined logic as "the Philosophy of the Laws of Thought *as expressed*" (page 126, emphasis added), and he began to stress that he was concerned with the *formal* laws of thought.

The comprehensiveness of language transcends our imagination. Boole declares this (page 41) when he says, "it is not true in thought that every operation can be performed upon every subject but it is possible in language to represent any operation as performed upon any subject, the laws which that operation obeys when possible in thought being still observed." In the

same sense, Boole prefers the term “notion” to the term “concept” “because the latter seems to indicate the formation of a mental picture or resemblance” (page 68).

This distancing from a psychological interpretation of all terms becomes understandable when Boole’s work on differential operators in the calculus is recalled (§6). He was also aware that the notion of limit was notoriously difficult to specify (pages 38-40). As a mathematician, Boole was well aware of its limitations and he always maintained this when he began to apply mathematics to logical problems. Thus Boole wrote to Penrose (pages 200-201; compare also page 175):

“It seems to be a law of human reason that we can in various instances affirm propositions ... respecting things which we can only picture or represent to ourselves as the limits of an indefinite process of abstraction. [...] [T]he object of all pure science are things which it would baffle all our efforts to represent by the power of imagination only and which can only be approved as the attainable limits of thought, but concerning which nevertheless the most vigorous of all propositions can be *affirmed* ...

We have now presented the factors that should be weighed up in an interpretation of Boole’s psychologism. The question as to whether these allow for a coherent whole must be left open. Suffice it to say that we feel we have shown Boole to be a great deal more interesting than legend leads one to suppose.

Part 3: Remarks on dating and editing the manuscripts

As was explained at the end of §2, the Royal Society collection has become rather disordered in the course of consultation and transcription over the decades. In recent times some valuable cataloguing has been effected by Panteki in the late 1980s and continued by Borner in the course of his selection and transcription for this edition; both scholars have placed their unpublished partial guides in the collection, to join one made by Alicia Boole and Falk. We provide readers of this edition with the required

information in the Textual Notes, where a general explanation is followed by the details required for each Chapter.

§22 Remarks on dating

Apart from the letters, no manuscripts are dated by Boole; thus chronology is a matter of conjecture. A few papers contain conclusive evidence by direct reference in the text such as “the present year 1848” or “this is written two years after *LT*”; the rest contains only hints. The two most important events regarding a chronology of the material are the publication of *LT* in 1854 and the marriage to Mary Everest the following year.

Mary helped George in the task of transcribing the manuscripts and in preparing them for the press. When she thought about depositing the unpublished works of her late husband she noted on the manuscript if it was known to her and therefore “later than 1855”. But this must not necessarily mean that the work originated after her marriage. It could well be that only the transcription was then made.

Regarding *LT*, the only clue for dating used here is an explicit reference in the text to this work. We did not consider the state of the theory presented in the paper when judging if it is prior or later than *LT*. Apart from the difficulties of such an undertaking there was usually better evidence to determine the date of the manuscript or the material was too fragmentary for such a consideration.

§23 Remarks on editing

We aim to retain the original spelling of Boole, including his sparing use of quotation marks. We keep his use of capitalization, although it sometimes looks a little arbitrary. We have rendered his underlinings as *italics*. In some cases he uses a vinculum instead of (brackets). This we also retained regardless of the danger of misunderstanding because in logic a vinculum often marks negation. Obvious mistakes of Boole have been corrected silently. Boole frequently uses “as” where today “just as” or “such as” would be used. Prescriptive modern commas have not been inserted where the meaning is clear.

Some of the manuscripts are in a unfinished state and/or written very hastily. It is not uncommon that references within the text are not supplied so that we only have an empty pair of brackets (). Where Boole left a totally

empty space for a later reference we complete the brackets silently for uniformity.

If a manuscript is in neat handwriting this is stated, but not otherwise. The transcriber is (with one exception, see chapter II) not known, for possible persons see §3 of the Introduction.

The footnotes of the manuscripts have no explicit reference in the text. Such references are supplied here silently. Footnotes belonging to the text itself are marked “*” and “†”. The figures 1, 2, 3, ... in different print from the basic text serve as reference to the Textual Notes at the end of the book.

Page numbers of the original version that appear in square brackets in the text thus “[XX.nn]” concern the marking of the then end-of-page. The “XX” identifies the document, “nn” the page number. Details can be found in the Textual Notes pp. 206ff.

Works mentioned by Boole are identified in the Textual Notes with the exception of his *LT*. In certain passages there would be simply too many references. All Greek expressions and texts appear in the Textual Notes in Latin script and where the meaning is not immediately obvious from the context they have been translated.

In contrast to the scrappiness and disorder of the collection, the writing itself is usually not too difficult to read. A typical folio is illustrated on the following page. It is p. 106 of Notebook 6 Logic, transcribed as Fragment 7, pages 189-190.

Acknowledgements

For access to the manuscripts, and much help from the library staff, we wish to thank the Royal Society of London. Our information on the Falks in §2 relied heavily on John Rollett, and on the letters to Russell held in the Russell Archives at McMaster University in Canada (archivist Kenneth Blackwell).

For help and advice we thank Desmond MacHale, Daniel D. Merrill, Maria Panteki, Volker Peckhaus and Adrian Rice. Andreas Bächli translated and explained all passages of the Boole material in Greek and Latin.

A special thanks goes to Elaine Lurf. She translated everything written by Gérard Bornet in German: i.e. part 2 of the introduction and much editorial material. She proofread the final copy.

The occupation of Bornet with Boole was part of a broader project on philosophical aspects of the mathematization of logic, which project was supported by the “Swiss National Foundation for the Advancement of

The whole theory of symbols is included in the following propositions

- 1st If a symbolical relation $=a$ is rightly interpreted, if when interpreted it expresses a proposition and that proposition is ^{correct} then the symbolical proposition is true
- 2nd If a symbolical $=a$ is rightly interpreted - if when so interpreted it expresses a proposition and that proposition is false - then the symbolical ~~proposition~~ ^{is false} is false
- 3rd If the interpretation is both the laws of combination hold ^{correct} and the laws of combination hold then the symbolical conclusion is correct
- 4th If a proposition is as true then a true proposition deduced from it & and from the necessary laws of thought is then a conclusion.

Here by interpretation is meant the putting for each symbol of its meaning. For a symbol $=a$ being correct is meant its being a logical consequence of some previous proposition. Thus a symbolical ~~conclusion~~ ^{premise} is correct because when the proposition it represents being true and the interpretation being false the given $=a$ results. A symbolical conclusion is correct when it flows from a symbolical premise & ~~interpret~~ ^{the laws of combination}.

Figure 8: A page from Notebook 6 Logic
(By permission of the President and Council of the Royal Society)
[transcription on pages 189-190]

Scientific Research” to whom thanks are due. We are also grateful for financial support from the “Swiss Society for Logic and the Philosophy of Science”, which made a third visit by Bornet to the library of the Royal Society in London possible.

Notes

- 1 Among biographical writings, the most distinguished is the book-length study *1985a* by the Cork mathematician Desmond MacHale; all further details should be first sought there.
- 2 Neil’s interests included logic and reasoning, which must have drawn him to Boole’s work; but it is quite unclear how, being based in Scotland, he came to know so much about Boole, who seems not to have referred to him.
- 3 In addition, the Stotts seemed to maintain an interest in matters algebraic, including connections to a Liverpool Mathematical Society. Mr. Stott was acknowledged in Ross *1905a*, a re-creation (more or less) of Arbogast’s methods which had been discussed before this society.
- 4 Oswald Toynbee Falk (1879-1972) was a stockbroker who later became involved in Treasury affairs and work with John Maynard Keynes (a younger friend of Russell, incidentally). For context see Skidelsky *1983a*, ch. 14.
- 5 The full extract reads as follows:

Pour aujourd’hui je ne veux répondre qu’à votre question touchant les mss. de Boole. D’abord, je suis très étonné d’apprendre que ces mss. se trouvent en d’autres mains que celles de Mrs. Boole, qui, m’a-t-on dit, conserve les publications de son mari, et à qui l’on doit s’adresser quand on veut se procurer certaines d’entre elles (c’est ce qui m’est arrivé quand j’ai voulu me procurer quelques mémoires de Boole: mon libraire les a demandés à Mrs. Boole). Quoi qu’il en soit, je suppose que l’authenticité des mss. en question est assurée. Je considérerais comme un grand honneur pour moi d’être chargé de l’édition de ces mss., à la condition qu’ils en valent la peine (c’est à dire qu’ils ne soient pas de simples brouillons ou copies de ce que Boole a publié), et à la condition qu’ils ne dépassent pas ma compétence (je pense notamment à ce qui regarde le calcul des probabilités, ou je vois pas encore bien clair, et où il est si facile des se tromper: Boole lui-même s’est trompé dans une question capitale de probabilité, selon M. Mac-Coll). Dans ce dernier cas, je demanderais au moins qu’on m’adjoignît un collaborateur scientifique. J’aimerais que ce fût vous, ou M. Whitehead: mais je présume que, si vous me proposez cette tâche, c’est parce que vous ne pouvez pas vous en charger, car vous vous en acquitteriez mieux que personne.

- 6 Alphonse Gratry (1805-1872), Oratorian Father and graduate of the *École Polytechnique* (a striking combination), published a two-volume *Philosophie — Logique* (1855 and later editions), conveying a mystical and deeply religious attitude to logic, stressing unity, and also covering philosophical questions in mathematics such as infinitesimals and the foundation of the calculus (Gratry *1944a*). According to Mary Boole, George also greatly liked his book, and the features just mentioned would have been congenial for him; but the manuscripts do not confirm her claim.

The contact between Mrs. Boole and Russell may have occurred through Una Birch (1876-1949), who became a well-known literary figure as Una Pope-Hennessy. She

wrote to Russell on 8 December 1904, thanking him for his views on Gratry and contrasting them with the enthusiasm of Mrs. Boole, who hoped for an English translation of *Logique*.

- 7 On the contributions of De Morgan and Jevons, see §13 and §14 below respectively.
- 8 In contrast to the great impact of Whately, the treatment of his work and its background in general histories of logic is miserable, and sometimes non-existent. A good start on him can be made with van Evra 1984a and Dessi 1988a. On logic teaching in Europe and the USA at that time, see Blakey 1851a, chs. 14-22.
- 9 It is possible that Boole and Whately met; in a letter of 15 December 1847 to the Rev. Edmund Larken (an old friend), Boole mentioned that John Graves, Professor of Jurisprudence at University College London and amateur mathematician, was trying to arrange a meeting to discuss *MAL* (University College Cork, Boole Papers, BP/1/223(11)).
- 10 For some reason Boole did not mention here his paper 1844a or cite Gregory, who had stated the left-hand trio in the paper 1839a. In a footnote he allowed (3) to permit “ $+n = +$ ”, which is also to be found in Gregory (1839b). In addition, Boole did not attribute the names of the first two laws to F. J. Servois (1767-1847), a follower of Arbogast.
- 11 On 26 January 1854 Boole wrote to De Morgan, reporting that he had asked his London publisher to send him a copy of *LT*. He also wished to send a copy to Sir John Herschel, and sought his address; and also commiserated with De Morgan on a loss in the family (presumably the death of his eldest daughter Elizabeth Alice the previous month). This letter was given in 1919 by De Morgan's grandson R. Campbell Morgan to the Bodleian Library, Oxford, where it now resides (Ms. Autograph d.14, fols 46-47): it is missing from the edition Smith 1982a of their correspondence.
- 12 The identity of Boole's financial friend is not known. Unfortunately no correspondence between the Macmillan Company and Boole is to be found in the publisher's archives in the British Library Additional Manuscripts, 54786-56035, 61894-61896. But the manuscript of *LT* survives in the Royal Society collection (U1-580), except for the last chapter, which is lost.
- 13 An interesting example is provided by John Hoppus (1789-1879), the founder Professor of Philosophy of Mind and Logic at University College, London. From 1855 until his retirement in 1867 he used *MAL* for a few questions in class examinations but alluded to *LT* only once. He was probably directed to *MAL* by his colleague professor De Morgan, whose own book *Formal Logic* had already been used by Hoppus for questions. His successor, G. Croom Robertson (1842-1892), stopped the practice (information from College Calendars, held in College Archives).
- 14 Rhees published several bits of this essay in Boole 1952a, 240-266; the fragmentation has not done it good service, so we give it complete here.
- 15 A textual puzzle attends this passage from Whately: he wrote ‘force’ rather than ‘form’. Presumably this is a misprint; however, it appears in every edition consulted, including the original encyclopaedia appearance (1823a, 209). Boole rendered it as ‘form’, as quoted here. ‘Force’ does appear sometimes in writings on logic of this period; for example, De Morgan used it to characterise the intensionality of a term (1864a, 105-106, 129).
- 16 In 1852 Boole wrote a long letter to Hamilton in defence of mathematics and its education; a copy survives in the Cork collection of Boole papers (BP/1/221(1), and was transcribed in MacHale 1985a, 133-134. No reply seems to be extant.
- 17 The word “ideology” comes from “idéologie”, proposed in 1796 by Destutt de Tracy to express the triad of the idea, the notion to which it refers, and the sign used to effect the reference. The quite different meaning which it carries today is due to the Marxists.

- 18 The significance of this passage has been noticed independently in Deakin 1996a, in a study of some of Boole's methods of solving differential equations. Note also the preface to the book.
- 19 Boole may have restricted $(x+x)$ to disjoint classes x and y to avoid multi-sets (as they are now called), collections to which a member may belong more than once. The unrestricted form does occur occasionally in Boole, as "signed heaps", to use the phrase of (Hailperin 1986a). The word "heap" comes from the introduction of multi-sets by the amateur British mathematician A. B. Kempe (1849-1922). His long paper (Kempe 1886a) was published by the Royal Society, but it interested only C. S. Peirce and, in the 1900s, the American philosopher Josiah Royce (1855-1916); for discussions, see (Vercelloni 1989a, ch. 1).
- 20 Venn could only handle up to four predicates with convex shapes; an ingenious algorithmic extension to any finite number is to be found in Edwards 1989a. On the differences between Euler's and Venn's methods, see Grattan-Guinness 1977a.
- 21 Moore 1993a, xxxvii refers to a letter of 13th June 1902 from Russell, who mentions that after the (putative) completion of his work on *The Principles of Mathematics* (Russell 1903a) he was undertaking a "systematic study of the great philosophers preparatory to my Logic". In this connection he got to work on *LT* (Russell Archives, ms. 220.010630). The study cannot have been very intensive, for in the space of a few days he also turned to De Morgan 1847a and Venn 1881a.
- 22 Passages where Boole refers to "the actual processes which take place in the mind of a person" (page 56) must be balanced against passages where he makes clear that the intellectual operations he is interested in are not concerned "with the nature of the individual object of thought" (page 67). Accordingly Boole leaves "the material and sensuous element" of concepts out of consideration (page 68). He also refers in his reply to the anticipated objection "that we do not actually reason thus" to an "ideal standard" and remarks, "that the actual performances of our nature" does not "in any case fully answer to its faculties and capacities" (page 111).
- 23 Here, it is true, Boole states in this connection the modernly assumed criterion that he thinks of "as fundamental those laws and principles from which all other general truths of science may be deduced, and into which they may all be again resolved." (*LT* 5). From today's standpoint the adjective "fundamental" would thereby become relative to an axiom system. The idea of such a relativity was, however, totally foreign at Boole's time, see also §19.
- 24 Frege (see §14) had Ernst Schröder rather than Boole in mind here (private communication from Volker Peckhaus). Husserl's earlier-quoted statement too was made in connection with a review of Schröder's work. This demonstrates how strongly Boole was considered only as a precursor of Schröder by a later generation.
- 25 Apart from the concepts of statement and argument, the concept of signs so especially central to formalism cannot be more closely defined without recourse to expressions such as "consciousness" or "thought".
- 26 Modernly put, Boole in actual fact muses on that which makes up structures without wishing to study structures as such as objects of investigation.
- 27 This lies behind the question discussed on page 72 as to whether logic is a noetic or ostensive science. The psychologism that is required to be rejected in formal logic sees logic as ostensive science; Boole, however, pronounces in favour of noetic science.