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The Maz'ya Anniversary Collection

Volume 1:

**On Maz'ya's work in functional analysis,
partial differential equations and applications**

**Jürgen Rossmann
Peter Takáč
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Vladimir Maz'ya

Maz'ya's portrait is by N. Singer

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Introduction

The contributions in this volume are dedicated to Vladimir G. Maz'ya and are partially based on talks given at the conference "Functional Analysis, Partial Differential Equations, and Applications", which took place at the University of Rostock from August 31 to September 4, 1998, to honour Prof. Maz'ya. This conference (a satellite meeting of the ICM) gave an opportunity to many friends and colleagues from all over the world to honour him. This academic community is very large. The scientific field of Prof. Maz'ya is impressively broad, which is reflected in the variety of contributions included in the volumes. Vladimir Maz'ya is the author and co-author of many publications (see the list of publications at the end of this volume), the topics of which extend from functional analysis, function theory and numerical analysis to partial differential equations and their broad applications. Vladimir G. Maz'ya provided significant contributions, among others to the theory of Sobolev spaces, the capacity theory, boundary integral methods, qualitative and asymptotic methods of analysis of linear and nonlinear elliptic differential equations, the Cauchy problem for elliptic and hyperbolic equations, the theory of multipliers in spaces of differentiable functions, maximum principles for elliptic and parabolic systems, and boundary value problems in domains with piecewise smooth boundaries. Surveys on Maz'ya's work in different fields of mathematics and areas, where he made essential contributions, form a major part of the present first volume of The Maz'ya Anniversary Collection. Other articles of this volume have their origin in the joint work with Maz'ya. (Most of the invited lectures of the Rostock conference are included in the second volume.)

V. G. Maz'ya's scientific commitment and creativity has influenced both his colleagues throughout the world and his research students. He is a master of creating a school of thought, and many young mathematicians owe their graduation to him. An active research group was created by him in Linköping where he works at present. In 1991 and 1994 he organized international conferences on applied and industrial mathematics in Linköping. The international reputation of Prof. Maz'ya is reflected in his membership in many editorial boards of scientific journals in Germany, Great Britain, Israel, Netherlands and Sweden.

In addition to the Rostock conference, a dedication to Vladimir Maz'ya on the occasion of his 60th birthday took place at the "International Symposium on Boundary Element Methods" organized by the Ecole Polytechnique in Palaiseau (Paris) in May 1998. The conference "Functional Analysis, Partial Differential Equations and Applications" which took place in Rostock a few months later was

jointly organized by the Department of Mathematics at the University of Rostock and the Weierstrass Institute of Applied Analysis and Stochastics in Berlin and was motivated by many scientific contacts of both institutions with Vladimir Maz'ya for many decades. In particular, it was motivated by the friendly relations which have grown over this period between Vladimir Maz'ya and the organizers of the conference. As a result of this long lasting cooperation many joint publications appeared.

My personal contact with V. G. Maz'ya goes back to my visit in the former city of Leningrad in 1969, upon his invitation. Joint interests in potential theory and the theory of elliptic differential equations of higher order have deepened our personal friendship and understanding. I remember my talk in the Mikhlin seminar very well. I was fascinated with the mathematical atmosphere in Leningrad at that time. It was characterized by such names as Smirnov, Mikhlin, Ladyzhenskaya, Havin, Solonnikov, and of course Maz'ya. The extremely stimulating discussions with V. G. Maz'ya have decisively influenced my further scientific research as well as the work of our research group, which was formed later in Rostock. That was the beginning of a personal friendship between us, which has lasted and strengthened through the decades, and I am very proud of that. I was not only impressed by V. G. Maz'ya's mathematical ideas but also by his fascinating personality. His interests are not limited to mathematics, but also extend to many other fields of life, such as art, especially of music and literature. My visits to Leningrad were always associated with impressive and enjoyable musical experiences, and I thank V. G. Maz'ya for that. I give him special thanks for his mathematical stimulus. Mathematicians in Rostock owe him a debt of gratitude as well as mathematicians throughout the world. It was a special honour for Rostock University to give Prof. Maz'ya an honorary doctorate in 1990. He takes his place together with such eminent scientists as Max Planck and Albert Einstein. With this honour and the conference of 1998, the university wants to thank V. G. Maz'ya for his support of mathematicians in Rostock.

The contributions in this volume show the extent of V. G. Maz'ya's research in an impressive way, his creativity, influence and worldwide recognition. The editors of this volume wish V. G. Maz'ya continued creativity and success. We also wish him and his family good health, happiness and stimulating and fruitful contacts, combined with a discussion of challenging mathematical problems.

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