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# Differentiable Operators and Nonlinear Equations

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Translated from the Russian by Mircea Martin

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# Introduction

We have considered writing the present book for a long time, since the lack of a sufficiently complete textbook about complex analysis in infinite dimensional spaces was apparent. There are, however, some separate topics on this subject covered in the mathematical literature. For instance, the elementary theory of holomorphic vector-functions and mappings on Banach spaces is presented in the monographs of E. Hille and R. Phillips [1] and L. Schwartz [1], whereas some results on Banach algebras of holomorphic functions and holomorphic operator-functions are discussed in the books of W. Rudin [1] and T. Kato [1].

Apparently, the need to study holomorphic mappings in infinite dimensional spaces arose for the first time in connection with the development of nonlinear analysis. A systematic study of integral equations with an analytic nonlinear part was started at the end of the 19th and the beginning of the 20th centuries by A. Liapunov, E. Schmidt, A. Nekrasov and others. Their research work was directed towards the theory of nonlinear waves and used mainly the undetermined coefficients and the majorant power series methods. The most complete presentation of these methods comes from N. Nazarov.

In the forties and fifties the interest in Liapunov's and Schmidt's analytic methods diminished temporarily due to the appearence of variational calculus methods (M. Golomb, A. Hammerstein and others) and also to the rapid development of the mapping degree theory (J. Leray, J. Schauder, G. Birkhoff, O. Kellog and others). These new methods were particularly attractive since they enabled the study of many classes of nonlinear equations, and therefore they were highly developed. (Important results were obtained by M. Krasnoselski, P. Zabreiko, V. Odinetz, Yu. Borisovich and B. Sadovski.) However, these new techniques retarded the development of specific methods for solving equations with an analytic nonlinear part. That is why in the sixties some mathematicians (P. Rybin, V. Pokornyi, M. Vainberg, V. Trenogyn and others) interested in the theory of integral equations and their applications returned to the Liapunov-Schmidt and Nekrasov-Nazarov analytic methods.

At the same time the theory of functions of one or several complex variables was enriched with more significant and subtle results. Parallel with these achievements, the first results on holomorphic mappings on infinite dimensional spaces appeared in the works of A. Cartan, R. Phillips, L. Nachbin, L.Harris, T. Suffridge, W.Rudin, M. Herve, E. Vesentini, J.-P. Vigue, P. Mazet, K. Goebel, and of many others.

We consider that it is now about the right time "to set a bridge" between nonlinear analysis and the theory of holomorphic mappings on infinite dimensional spaces. Of course, to this end it is necessary to put together results and techniques from the homology theory, sheaf theory, vector fields theory and from a lot of other modern theories in analysis — a task difficult to achieve within the limits of but one book. That is why we decided to start this vast project, by presenting only the theory of differentiable and holomorphic mappings on Banach spaces, as well as some prerequisites from functional analysis and topology.

In all chapters with the exception of Chapter 0 which has the character of a dictionary, we tried to give a complete account of definitions and proofs, and to make this book accesible not only to specialists, but also to students and to those engineers who are currently using the solutions of some specific integral and differentiable equations.

We conclude the work by mentioning the interesting relationship between the theory of holomorphic mappings and the theory of linear operators on spaces with indefinite metrics. More precisely, our last chapter is a brief exposition of the theory of spaces with indefinite metrics and of some relevant applications of the holomorphic mappings theory in this setting.

In closing, we draw our readers to a few technical points. Throughout the book we strove to use a uniform notation for objects of the same type. The most used notations are presented in Chapter 0. At the end of the book we give a list of some standard symbols, and also a subject index. We used the symbols "$\triangleleft$" and "$\blacktriangleright$" for the beginning and the end of a proof, respectively. The references in the text contain the name(s) of the author(s) followed by a number in brackets, which corresponds to the one in the reference list.

*THE AUTHORS*