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Nonlinear Multiobjective Optimization

A Generalized Homotopy Approach

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Dedicated to my parents

Preface

Real industrial systems are usually assessed by setting several objectives which are often competing with each other. Good compromise solutions are then looked for. The task of multiobjective optimization is to determine so-called efficient (or Pareto optimal) solutions which cannot be improved simultaneously with regard to all objectives.

The present book first gives a survey of the principles and classical methods of multiobjective optimization. Afterwards, the set of Pareto candidates is considered as a differentiable manifold, and a local chart is constructed which is fitted to the local geometry of this Pareto manifold. This opens up the possibility of generating new Pareto candidates by evaluating that local chart numerically. The generalized homotopy method thus developed has important advantages. It is capable of solving multiobjective optimization problems with an arbitrary number k of objectives, enables the generation of all types of Pareto optimal solutions and is able to produce a homogeneous discretization of the Pareto set. In the theoretical part of the book, the homotopy method is put on a sound mathematical basis by providing a necessary and sufficient condition for the set of Pareto candidates to form a $(k - 1)$ -dimensional differentiable manifold. The theoretical discussion is followed by a description of the numerical details of the proposed homotopy algorithm. Finally, by solving three multiobjective sample problems we demonstrate how this algorithm works in practice. Two of these problems originate in optimization applications within the configuration of industrial systems.

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