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Ginzburg-Landau Phase Transition Theory and Superconductivity

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Contents

Preface

The theory of complex Ginzburg-Landau type phase transition and its applications to superconductivity and superfluidity has been a topic of great interest to theoretical physicists and has been continuously and persistently studied since the 1950s. Today, there is an abundance of mathematical results spread over numerous scientific journals. However, before 1992, most of the studies concentrated on formal asymptotics or linear analysis. Only isolated results by Berger, Jaffe and Taubes and some of their colleagues touched the nonlinear aspects in great detail.

In 1991, a physics seminar given by Ed Copeland at Sussex University inspired Q. Tang, the co-author of this monograph, to study the subject. Independently in Munich, K.-H. Hoffmann and his collaborators Z. Chen and J. Liang started to work on the topic at the same time. Soon it became clear that at that time, groups of mathematicians at Oxford and Virginia Tech had already studied the subject for a couple of years. They inspired experts in interface phase transition problems and their combined effort established a rigorous mathematical framework for the Ginzburg-Landau system.

At the beginning Q. Tang collaborated with C.M. Elliott and H. Matano. When they met at the University of Sussex in the Summer of 1991, they discussed what kind of problems they should study. They found that the mathematical justification of the vortex structure and the relation between the applied magnetic field and the number of vortices were the two fundamental mathematical problems that the physicists failed to address, despite the long period they had been working on the subject. The trio also intended to look at a problem which was mathematically interesting: the limit behavior of the energy minimizers of the following Landau problem

$$\inf_{u|_{\partial B_1}=e^{in\theta}} \int_{B_1} \left(|\nabla u|^2 + \frac{1}{\varepsilon^2}(|u|^2-1)^2\right)dx$$

as $\varepsilon \longrightarrow 0$. Here B_1 is the unit ball in $\mathbb{R}^2$ and n is any integer. But during Matano's stay at Sussex, they didn't find a solution to the problem.

Back in Tokyo, Matano talked to H. Brezis, who visited Japan at the time, about the limit problem (in a Japanese café, according to H. Brezis), and that led to the famous work of [BBH 94]. Elliott, Matano and Tang justified the 2-d vortex structure rigorously.

By this time, many papers in this field treating modeling, asymptotic analysis and numerical analysis had already appeared. The research field is becoming even more active today with many new results being presented at every conference. However, to the best of our knowledge, the fundamental question of revealing the relation between the applied magnetic field and the number of vortices remains open. Hoffmann and his research team (as well as many other physicists and mathematicians around the world) carried out various numerical computations trying to gain insight into this relationship. The most spectacular numerical simulation we have seen so far was a 3-d film by H. Kappa who had the necessary superb computing facilities in his Argonne laboratory. Recently, H.-J. Bauer in his PhD thesis (from TU Munich) obtained progress in handling the problem with less computer power by using new concepts in scientific computing.

In this monograph, we try to collect the recent research results in the complex G-L theory with or without immediate applications to the theory of superconductivity. The purpose is to present as many mathematically sound results on various aspects of the PDE system as possible and provide a good reference for researchers who are interested in studying mathematical and physical problems in this field.

To fulfill this purpose, we include rigorous mathematical analysis, formal asymptotics as well as numerical analysis for the PDE system.

We start with some material on the physical background and point out some of the weaknesses in the modelling and theoretical studies of physicists. We then treat the mathematical scaling in a systematic way and analyze the implications on various limit problems. After that we address the mathematical foundation and formal asymptotic analysis of vortex motion. Then we concentrate on rigorous mathematics: we present results on existence, regularity and long time behavior of solutions and discuss the rigorous results on vortex location and law of motion. Furthermore, we look at various ways of deriving lower dimensional models from higher dimensional models and study rigorous results for the pinning of vortices.

We would like to thank the Alexander von Humboldt Foundation, the Center of Advanced European Studies and Research (caesar), Deutsche Forschungsgemeinschaft (DFG), Sussex University and the Technical University Munich for providing financial and moral support for the writing of this monograph.

We also would like to thank numerous colleagues for their interest in our work and in particular, Professor C.M. Elliott for his careful reading of the manuscript.

Q. Tang is also grateful to his wife, Xiaoyin, for her sincere and continuous support during his long absence from their beloved home in Brighton, England while staying in Bonn and Munich.