

## **Part VII**

# **Algorithms for QR Iterations and for Reduction to Hessenberg Form**

## Introduction to Part VII

The QR iteration method is a customary tool to compute all the eigenvectors of a matrix. In this part we develop fast QR iteration algorithms using quasiseparable representations of matrices. Unfortunately, in general the quasiseparable structure is not invariant under QR iterations. Thus one can obtain such algorithms only for special cases, such as Hermitian matrices, unitary matrices and their perturbations of small ranks.

In the first chapter we study explicit iteration methods. We show that the orders of lower quasiseparable generators of matrices do not increase in the iteration process. This allows one to obtain fast algorithms for the cases when one can reconstruct upper quasiseparable generators of a matrix in terms of lower ones and diagonal entries. This is valid for the aforementioned Hermitian and unitary matrices and their perturbations of small ranks.

An effective method for solving the eigenvalue problem is a preliminary reduction of a matrix to a unitary similar one in upper Hessenberg form. The subsequent QR iterations for upper Hessenberg matrices can be performed in an efficient way. The reduction to Hessenberg form method for matrices with quasiseparable representations is studied in the second chapter.

In the last chapter we study the implicit QR iteration methods for upper Hessenberg matrices obtained via the reduction step.