Part VI

Divide and Conquer Method for the Eigenproblem

Introduction to Part VI

The method for the eigenvalue decomposition we study here consists of two steps, the divide step and the conquer step. Accordingly, the first two chapters of the present part are dedicated each to one of these two steps. In the third chapter we apply and use quasiseparable generators for the divide and conquer method for Hermitian matrices, especially order one quasiseparable Hermitian matrices. This method is well known for tridiagonal matrices, which are an example of quasiseparable matrices of order one. The last section is devoted to the divide and conquer method for unitary Hessenberg matrices.

The divide step consists in splitting a single problem into two smaller independent problems with size roughly half the size of the original problem. This is done recursively, until the obtained problems are of a convenient size, which is small enough so that they can be solved by standard techniques. We perform the divide step for any square matrix by using its quasiseparable representation and we obtain the quasiseparable generators of the resulting two smaller matrices. The order of the quasiseparable generators does not increase in the process.

In the case when the initial matrix belongs to one of certain important classes of matrices, like Hermitian, tridiagonal matrices, companion matrices, or unitary Hessenberg matrices, we also show that the two smaller matrices also belong to the same class.

After the division step of the algorithm is completed and the eigenstructure of the smallest matrices has been found, we perform the conquer step, in which the division tree is climbed back and we obtain the eigenstructure of a larger matrix A from knowledge of the eigenstructure of two smaller matrices B and C. To do this we need to compute the eigenstructure of a small sized matrix function with size equal to the order of perturbation. While performing the conquer step we impose increasingly restrictive conditions in order to obtain more results. Complete algorithms for computing the eigenvalue decomposition are obtained for quasiseparable of order one Hermitian matrices and for unitary upper Hessenberg matrices.