

Operator Theory: Advances and Applications

Volume 235

Founded in 1979 by Israel Gohberg

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Separable Type Representations of Matrices and Fast Algorithms

Volume 2
Eigenvalue Method

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ISSN 0255-0156 ISSN 2296-4878 (electronic)
ISBN 978-3-0348-0611-4 ISBN 978-3-0348-0612-1 (eBook)
DOI 10.1007/978-3-0348-0612-1
Springer Basel Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013951156

Mathematics Subject Classification (2010): 15A18, 65F15, 65H04

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Printed on acid-free paper

Springer Basel is part of Springer Science+Business Media (www.birkhauser-science.com)

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Introduction

The book. This book contains a systematic theoretical and computational study of several types of generalizations of separable matrices. It is related to semiseparable, quasiseparable, band and companion representations of matrices. For them their natural parameters, called generators, are analyzed and algorithms are expressed in terms of generators.

In this volume we apply results and methods developed in Volume 1 to study in detail the eigenstructures of matrices with quasiseparable representations and to develop fast eigenvalue algorithms. A large number of illustrative examples are provided. The present volume contains Parts V–VIII.

Description of parts. The volume addresses the eigenvalue problem for matrices with quasiseparable structure. In Part V, for matrices with quasiseparable representation of the first order, we perform a detailed study of the properties of the characteristic polynomials of principal leading submatrices, the structure of eigenspaces, and the basic methods to compute eigenvalues. Part VI is devoted to the divide and conquer method; the main algorithms are derived also for matrices with quasiseparable representation of order one. In Part VII, for some classes of matrices with quasiseparable of any order representations, we study the QR iteration method. This method is used in Part VIII in order to get a fast solver for the polynomial root finding problem.

The first volume consists of Parts I–IV. The titles are as follows. Part I: Basics on separable, semiseparable and quasiseparable representations of matrices; Part II: Completion of matrices with specified band; Part III: Quasiseparable representations of matrices, descriptor systems with boundary conditions and first applications; Part IV: Factorization and inversion.