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Constrained Optimization and Optimal Control for Partial Differential Equations

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Contents

Introduction

Solving optimization problems subject to constraints involving distributed parameter systems is one of the most challenging problems in the context of industrial, medical and economical applications. In particular, in the design of aircraft, 'moving bed' processes in chemical engineering, crystal growth etc., the transition from model-based numerical simulations to model-based design and optimal control is crucial. All of these applications involve modeling based on partial differential equations (PDEs), and can be classified as form and topology optimization problems or optimal control problems in an infinite-dimensional setting.

For the treatment of such problems the interaction of optimization techniques and numerical simulation is crucial. After proper discretization, the number of optimization variables varies between 10^3 and 10^{10}. It is only very recently that the enormous advances in computing power have made it possible to attack problems of this size. However, in order to accomplish this task it is crucial to utilize and further explore the specific mathematical structure of prototype applications and to develop new mathematical approaches concerning structure-exploiting algorithms, adaptivity in optimization, and handling of control and state constraints. It is necessary to combine numerical analysis of PDEs and optimization governed by prototype applications so that the most recent activities in the fields are merged and further explored.

On the one hand, new models, and along with them new mathematical challenges, also require basic theoretical research. The synergy between theoretical developments, numerical analysis and its realizations make the group of contributors involved in the program, and hence these research notes, a unique collection of ideas and discussions. The organization of this volume is as follows: there are five topics, namely,

- Constrained Optimization, Identification and Control
- Shape and Topology Optimization
- Model Reduction
- Discretization: Concepts and Analysis
- Applications

which are covered by overview articles complemented by short communications. This gives the opportunity to provide an introduction to each topical area along with a specialization. Below we provide an executive summary for each topical area and then give a more detailed description of the actual content.

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G. Leugering, Spokesman

Constrained Optimization, Identification and Control

Several projects of the priority program consider the development of novel algorithmic concepts for the numerical solution of PDE constrained optimization, identification, and optimal control problems. Semi-smooth Newton techniques are developed and analyzed for optimization problems with PDE- and control constraints as well as for time-dependent variational equations. In particular, optimization techniques that use multigrid methods, adaptivity, model reduction, and structure-exploiting algorithmic differentiation are proposed for complex time-dependent optimal control problems with applications to fluid dynamics and PDAEs. Moreover, the development of one-shot techniques based on iterative solvers for the governing PDE-solver is considered. Furthermore, analytical and algorithmic advances in challenging applications such as the optimal control of networks of hyperbolic systems, optimum experimental design for PDE-constrained problems, and the feedback stabilization of incompressible flows are presented.

M. Ulbrich, S. Ulbrich and A. Griewank

Shape and Topology Optimization

Here, novel algorithmic approaches towards the practically highly important class of shape and topology optimization are presented and also generalized to a stochastic setting. Allen-Cahn and Cahn-Hilliard-formulations of topology optimization problems based on the homogenization approach are investigated. Free boundary value problems based on the Poisson equation are treated by the use of shape calculus of first and second order. Furthermore, new algorithmic schemes are developed and analyzed for parametric and non-parametric shape optimization in the context of flow problems.

V. Schulz

Model Reduction

Within this special volume, recent developments are presented in the adaptive discretization of elliptic or parabolic PDE-constrained optimization problems. The possibility of additional pointwise control or state constraints is included. Particular emphasis is put on automatic model reduction by adaptive finite element discretization for the accurate computation of a target quantity ("goal-oriented adaptivity").

R. Rannacher

Discretization: Concepts and Analysis

Within the priority program, recent trends and future research directions in the field of tailored discrete concepts for PDE constrained optimization with elliptic and parabolic PDEs in the presence of pointwise constraints are addressed. This includes the treatment of mesh grading and relaxation techniques, of adaptive a posteriori finite element concepts, and of a modern treatment of optimal control problems with parabolic PDEs. Tailoring discrete concepts is indispensable for a proper and effective treatment of complex model problems and applications.

M. Hinze

Applications

As explained above, the entire project, while mathematically oriented, ultimately aims at and is justified by challenging applications. The participating research groups are convinced that the relation between modern mathematics and applications is a two-way interaction where both sides profit from each other: new mathematics is created by pushing the boundary of applications that can be tackled. In this sense, several applications in the domain of optimization and control with partial differential equations are provided here. They range from the nano-scale to the macro-scale. The areas of application are chosen from electrical, mechanical and chemical engineering as well as from medical and biological sciences.

S. Engell and G. Leugering