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Simplicial Methods for Operads and Algebraic Geometry

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Foreword

This book is an introduction to two higher-categorical topics in algebraic topology and algebraic geometry relying on simplicial methods. It is based on lectures delivered at the Centre de Recerca Matemàtica in February 2008, as part of a special year on Homotopy Theory and Higher Categories.

Ieke Moerdijk's lectures constitute an introduction to the theory of *dendroidal sets*, an extension of the theory of simplicial sets designed as a foundation for the homotopy theory of operads. The theory has many features analogous to the theory of simplicial sets, but it also reveals many new phenomena, thanks to the presence of automorphisms of trees. Dendroidal sets admit a closed symmetric monoidal structure related to the Boardman–Vogt tensor product. The lecture notes develop the theory very carefully, starting from scratch with the combinatorics of trees, and culminating with a model structure on the category of dendroidal sets for which the fibrant objects are the inner Kan dendroidal sets. The important concepts are illustrated with detailed examples.

The lecture series by Bertrand Toën is a concise introduction to *derived algebraic geometry*. While classical algebraic geometry studies functors from the category of commutative rings to the category of sets, derived algebraic geometry is concerned with functors from simplicial commutative rings (to allow derived tensor products instead of the usual ones) to simplicial sets (to allow derived quotients instead of the usual ones). The central objects are derived (higher) stacks, which are functors satisfying a certain up-to-homotopy descent condition. The lectures start with motivating examples from moduli theory, to move on to simplicial presheaves and algebraic (higher) stacks; next comes the homotopy theory of simplicial commutative rings, and finally everything comes together in the notion of derived (higher) stack. Some proofs are given as exercises that involve consulting the literature.

Both lecture series assume a working knowledge of Quillen model categories. For Toën's lectures, some background in algebraic geometry à la Grothendieck is also necessary.

We are very thankful to the CRM for hosting the advanced course as well as the whole research programme on Homotopy Theory and Higher Categories. The former director Manuel Castellet and his successor Joaquim Bruna made this possible. The CRM secretaries were much more than helpful at all times. We are

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Carles Casacuberta and Joachim Kock

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