Part I

Lectures on Dendroidal Sets

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Preface

The theory of dendroidal sets forms a new attempt to give a combinatorial theory of topological structures which involve operations with one output and multiple inputs, such as the theory of operads. The theory seems to be developing in a way similar to that of simplicial sets, although there are some noticeable differences. For example, the combinatorics of finite linear orders is replaced by that of finite rooted trees, which causes some technical complications. Other important differences are that the cartesian product of simplicial sets is replaced by a closed symmetric monoidal structure, which no longer commutes with the nerve construction. Related to this, the category Ω which takes the place of the simplicial index category Δ does not have a terminal object, and in fact the homotopy type of its classifying space $B\Omega$ is as yet unknown.[1] Finally, an important gap (at least, up to now) in the theory is the lack of a suitable geometric realization functor which is compatible with this monoidal structure —a functor from dendroidal sets into the category of topological spaces or into a closely related category.

The notes which you have in front of you are a faithful presentation of the lectures that I gave in the context of an advanced course on Simplicial Methods in Higher Categories, as part of a special year on Homotopy Theory and Higher Categories at the CRM in Barcelona. In these lectures, I have reported on recent research, done partly in collaboration with Clemens Berger, Denis-Charles Cisinski and Ittay Weiss, as listed in the references. The lectures owe a lot to their insights, and I apologize for what they undoubtedly consider as an inadequate representation of their ideas about operads and dendroidal sets.

I would like to thank the people involved in the organization of the CRM special year, in particular the coordinators Carles Casacuberta and Joachim Kock, for giving me the opportunity to expose the theory of dendroidal sets, and for providing me with such an inspiring and active audience. But above all, I am immensely grateful to Javier Gutiérrez, who helped me turn scattered notes of the lectures and bits from unpublished papers into a coherent text.

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[1] *Added in proof:* I have shown in the meantime that $B\Omega$ is contractible.