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Bayesian and High-Dimensional Global Optimization

 Springer

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Preface

Global optimization (GO) is a broad and active field of research including mathematical analysis of problems, development of algorithms and software, and applications to various real-world problems. Many approaches have been proposed to tackle different subclasses of GO problems. In this book we consider the algorithms aimed at two challenging classes of GO problems usually specified as black-box expensive and high-dimensional. We mainly focus on two approaches: the Bayesian approach and high-dimensional random or pseudo-random search.

Interests in Bayesian and high-dimensional GO have been growing fast during last few decades. The development of algorithms was stimulated by new applications especially in optimal engineering design where optimization problems are characterized as black-box and expensive. Important, recently emerged application areas are related to computer experiments and machine learning. The theoretical investigation in Bayesian and high-dimensional GO was carried out in parallel with the development of algorithms. In the present monograph, some of the original ideas are presented as well as their further developments, discussing also challenges and unsolved problems.

The book has three chapters. Chapter 1 starts by considering properties of high-dimensional cubes and balls, then various aspects of uniformity and space-filling are discussed. It is demonstrated that good uniformity of a set of points does not have to imply its good space-filling. Special attention in Chap. 1 is devoted to covering, which is a very important criterion of space-filling. In particular, the concept of weak covering is introduced, where only a large part of a feasible domain has to be covered by the balls with centers at given points, rather than the full domain as in the standard covering. In Chap. 2, we present our recent work in Bayesian approach to continuous non-convex optimization. A brief review precedes the main results to have our work presented in the context of challenges of the approach. Chapter 3 is devoted to global random search (GRS). It is not the aim of this chapter to cover the whole subject; we only make some potentially important notes on algorithms of GRS in continuous problems, mostly keeping in mind the use of such algorithms in reasonably large dimensions.

In the process of writing the book, we have spent long hours discussing its various parts, but responsibility for different chapters is not shared. Anatoly takes full responsibility for Chaps. 1 and 3 while Antanas is responsible for the material of Chap. 2.

Overall, the book is written for a wide circle of readers and will hopefully be appreciated by those interested in theoretical aspects of global optimization as well as practitioners interested mostly in the methodology. All those interested in applications of global optimization can also benefit from the book.

We are very grateful to both referees of the book for very detailed and very helpful comments. We are also very grateful the editors of the *SpringerBriefs in Optimization* and especially to Prof. Panos Pardalos for his encouragement of our project. We thank Jonathan Gillard, Jack Noonan, and Andrey Pepelyshev (Cardiff) for helping to improve English and maths in many parts of the book. The research by A. Žilinskas was funded by a grant (No. S-MIP-17-61) from the Research Council of Lithuania.

Cardiff, UK
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October 2020

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