

Mechanical Vibrations

Tony L. Schmitz • K. Scott Smith

Mechanical Vibrations

Modeling and Measurement

Second Edition



Springer

Tony L. Schmitz
Mechanical, Aerospace, and
Biomedical Engineering
University of Tennessee, Knoxville
Knoxville, TN, USA

K. Scott Smith
Manufacturing Demonstration Facility
Machining and Machine Tools Research Group
Oak Ridge National Laboratory
Knoxville, TN, USA

ISBN 978-3-030-52343-5 ISBN 978-3-030-52344-2 (eBook)
<https://doi.org/10.1007/978-3-030-52344-2>

© Springer Nature Switzerland AG 2012, 2021

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors, and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To our children, Jake, BK, Kellye, and Kyle

Preface

In this second edition textbook, we describe essential concepts in the vibration analysis of mechanical systems. The book incorporates the fundamentals of modal analysis, beam theory, and finite element analysis, as well as the required mathematics and experimental techniques, into a unified framework that is written to be accessible to undergraduate students, researchers, and practicing engineers alike. This book is based on undergraduate courses in mechanical vibrations that we have previously offered and developed the text to be applied in a traditional 15-week course format. It is appropriate for undergraduate engineering students who have completed the basic courses in mathematics (through differential equations) and physics and the introductory mechanical engineering courses including statics, dynamics, and mechanics of materials.

We organized the book into ten chapters. The chapter topics are summarized here.

- Chapter 1—We introduce the types of mechanical vibrations, damping, and periodic motion.
- Chapter 2—We explore topics in single degree of freedom free vibration, including the equation of motion, the damped harmonic oscillator, and unstable behavior.
- Chapter 3—We introduce single degree of freedom forced vibration and discuss the frequency response function, rotating unbalance, base motion, and the impulse response.
- Chapter 4—We extend the analysis in Chap. 2 to consider two degrees of freedom free vibration. This includes the eigensolution for the equations of motion and modal analysis.
- Chapter 5—We extend the analysis in Chap. 3 to consider two degrees of freedom forced vibration. We describe complex matrix inversion, modal analysis, and the dynamic absorber.
- Chapter 6—We analyze model development by modal analysis. This incorporates the peak picking approach for identifying modal parameters from a system frequency response measurement and mode shape measurement.

- Chapter 7—We describe frequency response function measurement techniques. Impact testing is highlighted.
- Chapter 8—The topic of this chapter is continuous beam modeling. We develop closed-form frequency response function expressions for transverse beam vibration, torsion vibration, and axial vibration of beams.
- Chapter 9—We provide an introduction to finite element analysis. Beam models for axial and transverse vibration are derived and implemented; this includes the mass and stiffness matrices. Examples are provided to evaluate model convergence as elements are added. This chapter is new to the second edition.
- Chapter 10—We introduce the concept of receptance coupling, where frequency response functions (receptances) are coupled to predict assembly dynamics.

To demonstrate and unify the various concepts, the beam experimental platform (BEP) is used throughout the text. Engineering drawings for the BEP are included in Appendix A, so that instructors can provide their own demonstrations in the classroom. Additionally, MATLAB[®] programming solutions are integrated into the text through many numerical examples.

Special features of the book include: (1) MATLAB[®] MOJO code examples; (2) *By the Numbers* numerical solutions; (3) problems and solutions, including MATLAB[®] code; (4) non-mathematical *In a Nutshell* explanations that summarize selected concepts in layman's terms; and (5) discussions and numerical examples of model uncertainty.

We conclude by acknowledging the many contributors to this text. They naturally include our instructors, colleagues, collaborators, and students. Among these, we would like to particularly recognize the contributions of J. Tlustý, M. Davies, T. Burns, J. Pratt, and H.S. Kim.

Knoxville, TN
Knoxville, TN
April 2020

Tony L. Schmitz
K. Scott Smith

Contents

1	Introduction	1
1.1	Mechanical Vibrations	1
1.2	Types of Vibrations	2
1.2.1	Free Vibration	2
1.2.2	Forced Vibration	3
1.2.3	Self-Excited Vibration	4
1.3	Damping	6
1.4	Modeling	6
1.5	Periodic Motion	10
	Exercises	25
	References	28
2	Single Degree of Freedom Free Vibration	29
2.1	Equation of Motion	29
2.2	Energy-Based Approach	40
2.3	Additional Information	46
2.3.1	Equivalent Springs	46
2.3.2	Torsional Systems	48
2.3.3	Nonlinear Springs	49
2.4	Damped Harmonic Oscillator	51
2.4.1	Viscous Damping	51
2.4.2	Coulomb Damping	52
2.4.3	Solid Damping	52
2.4.4	Damped System Behavior	52
2.4.5	Underdamped System	54
2.4.6	Damping Estimate from Free Vibration Response	65
2.4.7	Damping Estimate Uncertainty	68
2.5	Unstable Behavior	70
2.5.1	Flutter Instability	71
2.5.2	Divergent Instability	75

2.6	Free Vibration Measurement	81
	Exercises	83
	References	87
3	Single Degree of Freedom Forced Vibration	89
3.1	Equation of Motion	89
3.2	Frequency Response Function	90
3.3	Evaluating the Frequency Response Function	95
3.4	Defining a Model from a Frequency Response Function Measurement	111
3.5	Rotating Unbalance	115
3.6	Base Motion	119
3.7	Impulse Response	124
	Exercises	128
	References	132
4	Two Degree of Freedom Free Vibration	133
4.1	Equations of Motion	133
4.2	Eigensolution for the Equations of Motion	135
4.3	Time-Domain Solution	145
4.4	Modal Analysis	152
	Exercises	166
	References	172
5	Two Degree of Freedom Forced Vibration	173
5.1	Equations of Motion	173
5.2	Complex Matrix Inversion	175
5.3	Modal Analysis	182
5.4	Dynamic Absorber	191
	Exercises	199
6	Model Development by Modal Analysis	205
6.1	The Backward Problem	205
6.2	Peak Picking	205
	6.2.1 Single Degree of Freedom	205
	6.2.2 Two Degrees of Freedom	208
6.3	Building the Model	209
6.4	Peak Picking for Multiple Degrees of Freedom	220
6.5	Mode Shape Measurement	223
6.6	Shortcut Method for Determining Mass, Stiffness, and Damping Matrices	229
	6.6.1 Linearized Pendulum	233
	6.6.2 Automobile Suspension Model	237
	Exercises	242
	References	253

- 7 Measurement Techniques** 255
 - 7.1 Frequency Response Function Measurement 255
 - 7.2 Force Input 256
 - 7.3 Vibration Measurement 258
 - 7.3.1 Capacitance Probe 258
 - 7.3.2 Laser Vibrometer 259
 - 7.3.3 Accelerometer 260
 - 7.4 Impact Testing 265
 - 7.5 Modal Truncation 274
 - Exercises 280
 - Reference 282

- 8 Continuous Beam Modeling** 283
 - 8.1 Beam Bending 283
 - 8.2 Transverse Vibration Equation of Motion 288
 - 8.3 Frequency Response Function for Transverse Vibration 289
 - 8.3.1 Fixed-Free Beam 290
 - 8.3.2 Free-Free Beam 295
 - 8.4 Solid Damping in Beam Models 299
 - 8.5 Rotation Frequency Response Functions 305
 - 8.6 Transverse Vibration FRF Measurement Comparisons 308
 - 8.6.1 Fixed-Free Beam 308
 - 8.6.2 Free-Free Beam 310
 - 8.6.3 Natural Frequency Uncertainty 312
 - 8.7 Torsion Vibration 313
 - 8.8 Axial Vibration 315
 - 8.9 Timoshenko Beam Model 319
 - Exercises 320
 - References 324

- 9 Finite Element Introduction** 325
 - 9.1 Introduction 325
 - 9.2 Axial Element 326
 - 9.3 Transverse Element 349
 - Exercises 363
 - References 365

- 10 Receptance Coupling** 367
 - 10.1 Introduction 367
 - 10.2 Two Component Rigid Coupling 367
 - 10.3 Two Component Flexible Coupling 372
 - 10.4 Two Component Flexible-Damped Coupling 380
 - 10.5 Comparison of Assembly Modeling Techniques 381
 - 10.5.1 Modal Analysis 383
 - 10.5.2 Complex Matrix Inversion 385
 - 10.5.3 Receptance Coupling 386

- 10.6 Advanced Receptance Coupling 390
- 10.7 Assembly Receptance Prediction 396
 - 10.7.1 Free-Free Beam Coupled to Rigid Support 397
 - 10.7.2 Free-Free Beam Coupled to Fixed-Free Beam 403
 - 10.7.3 Comparison Between Model and BEP Measurement 408
- Exercises 412
- References 414

- Appendix A: Beam Experimental Platform 415**
- Appendix B: Orthogonality of Eigenvectors 417**
- Index 421**