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Understanding Complex Systems

Founding Editor: S. Kelso

Future scientific and technological developments in many fields will necessarily depend upon coming to grips with complex systems. Such systems are complex in both their composition – typically many different kinds of components interacting simultaneously and nonlinearly with each other and their environments on multiple levels – and in the rich diversity of behavior of which they are capable.

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Chaos

The World of Nonperiodic Oscillations

Springer

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Preface

To chaos, I came like the virgin to the child (to quote a proverb). In 1975, I had given a talk in Vienna on biological clocks when Art Winfree told me that I had given better talks before—I, therefore, should take a look at "chaos." I had no idea what he meant, but 5 weeks later a big folder arrived with all papers and preprints existing up to that day on the subject of chaos in the modern sense, with the accompanying message that I should make something out of it since he momentarily lacked the time to do it himself. This was so overwhelming a gift that I felt obliged to try and generate a chemical reaction—kinetic analog with corresponding nonnegative rate equations as told to do. This proved to be a task vastly overtaxing my abilities. But I was in the bind to have to deliver for a good friend. So despair turned into chutzpah, and I tried gluing together two flows on a letter-Z-shaped sheet, one level laid above the other, with jumps at the edges, and with the height difference causing a shift of the two-dimensional oscillator assumed valid on the one sheet. It couldn't work. And unexpectedly the outcome was even simpler than the Lorenz attractor, which I was meant to reproduce in an abstract reaction system. So it was all Art's fault, and my attempt not to leave unreciprocated the tremendous effort made by a close friend.

The next gift from heaven was the presence of a modern analog computer in the department in which I worked. I had been sent to a computer course offered by the *Electronic Association Inc.* (EAI)—the biggest analog computer company at the time—which in fact helped me lose my inhibitions also toward the Dornier machine that was subsequently bought by our department. So "playing with equations" had become an available realistic option. Since nonlinearities are a notorious problem with analog computers, interactively simplifying the theoretical ("singular-perturbation") equations written down first in the two-sheet model as mentioned, was a natural option as well as a necessity. This lead to the simple equations that subsequently got recorded in the "chaos" movie which my wife, who had grown enthusiastic about the new reality, produced jointly with a coworker, Thomas Wiehr, from her lab in the Medical Policlinic of the University of Tübingen. The "chaos" movie also has sound (the third variable recorded on the soundtrack). Two years later, "hyperchaos" got recorded in the same fashion once more, which

likewise can be watched and listened-to on the "Chaos" movie of 1976/79 that is on YouTube.[1]

The "sound of chaos" as well as that of hyperchaos turned out to be very familiar from daily life. The three speeds with which it was recorded (available on the Dornier machine) showed that snoring and a hoarse voice and piercing noises—but also wind-driven oscillations in another part of the body—were all chaotic. When there is a tiny lump on one of the two lips of the larynx, for example, spoken language necessarily becomes "hoarse," that is, chaotic. Nature is replete with three- and four-variable nonlinear dynamical systems of dissipative type. Also the irregular puttering of a motorcycle or car in the idling mode belongs here, not to mention cardiac irregularities. An abstract of ours written jointly with Herbert D. Landahl, a friend and former coworker of Nicholas Rashevsky, about cardiac arrhythmia came out of this work as well, presented in Japan in 1979. Another implication was endocrine chaos, discovered simultaneously independently by Colin Sparrow in England.

Many decades later, the new fundamental science of Cryodynamics, sister of deterministic Thermodynamics, would trigger a bet by the whole scientific community, on its being invalid because a multibillion dollar experiment based on the opposite assumption proved to have potentially deleterious consequences for the whole world. From the point of view of chaos theory, such counterintuitive "re-injections" make perfect sense. Society is also a dissipative dynamical system itself given over to unpredictable bifurcations and risks. Imagine the historical fact of warfare being adopted in irregular intervals in history.

Misunderstandings can thus be classified as generating dangerous societal developments. The new science of deductive brain theory, implicit in the deductive biology invented in dialog with Konrad Lorenz in 1966, subsequently allowed for the discovery of the interactive "smile explosion." It can be understood as arising between two brain equations or, rather, their carriers. Gregory Bateson was the first person to understand and appreciate the mechanism in 1975, but he was also the last. It explains the creative misunderstanding of the suspicion of benevolence being at work on the other side, invented by the toddler. So the small child becomes a carrier of a manifold, once more. The spontaneous invention of personhood, by the suspicion of benevolence being present on the other side is nothing but another analog to the two sheets that creatively allow for chaos in a tangle of deterministic operations. Here, two mirror-competent autonomous optimizers with cognition invent the conjectured existence of a benevolent intention being present on the other side. It is not chaos, and brains are not manifolds, but the analogy is clear. This particular application of chaotic visualization would enable the formulation of the "personogenetic bifucation" in close analogy to the two manifolds involved in chaos

[1] https://www.youtube.com/watch?v=Tmmdg2P1RIM.

generation. This "sociological" realization of the letter-Z-shaped chaos-genesis is still very much in its spirit. Only that the invention of the suspicion of benevolence being encountered generates the exact opposite of chaos—unlimited trust—much as between Art and Christophe and me. But I now do better stop.

Tübingen, Germany Otto E. Rössler
November 2019

When I started my Ph.D. thesis, I started quite early to read Otto's papers, not the two well-known ones, but those full of beautiful hand-drawn pictures as shown in Fig. 2.5, p. 27. I always enjoy when I can draw a picture explaining the concepts that I am manipulating in my researches. Otto's drawings helped me a lot to understand branched manifold and their relationships with first-return map to a Poincaré section. I then worked for more than 10 years almost exclusively on the topological characterization of chaotic attractors, in particular with Robert Gilmore from Drexel University (Philadelphia).

In the early 2000s, I was working with some biologists from Rouen (Camille Ripoll, Janine Guespin, and Michel Thellier). They introduced me to René Thomas (Brussels) with who I enjoyed many exchanges. Unfortunately, we were never successful to maturate sufficiently our results related to a relationship between René's feedback circuits and branched manifolds for publishing them. At that time, René was in contact with Otto and recommended me to him. Otto kindly accepted an invitation to give a talk in Rouen. During the lunch after the talk, we had an amazing discussion about almost everything. I enjoyed so much the easiness with which Otto was flowing in the world of ideas. We kept in touch since that time.

During one of my visits in Tübingen, Otto's wife, Reimara, mentioned the existence of a manuscript written in the early 1980s which was never published. Otto submitted it to Springer which rejected it! It would have been the second book about chaos theory, the first one was written by Igor Gumowski and Christian Mira and published (hélas en français) in 1980.[2] Otto never submitted its manuscript to another publisher… It took about one year for seeing this manuscript, the time that Reimara recovered it, and that I returned to Tübingen for another visit. And then… an amazing journey in Otto's mind, full of astonishing discoveries. So many things were still new and not expressed with so much clarity.

[2] I. Gumowski & C. Mira, *Dynamique chaotique: transformations ponctuelles, transition ordre-désordre*, Cepadues Editions, 1980.

Otto and Christophe enjoying in digging in Otto's archives. Tübingen, 2010.

I immediately proposed to edit the manuscript, retyping everything, (re)computing all the figures. My target was to leave the text as much as possible in its original form and just to provide high-resolution pictures. I could say that for 90% of the pages, there is change neither in the text nor in the simulations which were obtained in one shot from the information reported in the manuscript. In some cases, the pictures were only drawn by hand with parameter values and initial conditions reported: even for these ones, the computed pictures were matching with the drawn ones. Unfortunately, for a few pictures, I was unable to reproduce them. With Otto, we tried to dig in his archives, sometimes we found the missing information, and sometimes not. In these last cases, it turned to be complicated to construct examples as initially imagined by Otto and we had to make some modifications of the text. This is limited to the two sections devoted to the blue sky catastrophe, and to the two sections where chaos in the Bonhoeffer–van der Pol equations is discussed. We tried to remain as much as possible close to the initial approach. During the edition of this manuscript, I added some references (marked with a "*") and some footnotes, particularly when Otto anticipated some results which came a few years later.

I wish to thank Jürgen Kurths who supported me for completing this project and helped for having this book finally published with Springer. He invited us to add an appendix about some realizations of chaos in the real world. They mostly came from my own researches performed in collaboration with many collaborators. I started by adding Otto's own contribution in finding chaos in the Belousov–Zhabotinski reaction and the one provided by one of Otto's great friends, John Hudson (Charlottesville) with who I collaborated—before meeting Otto—for investigating experimental data from an electrodissolution. I wish to thank Thomas Klinger for having provided the data from a plasma experiment. With Jean Maquet (Rouen), Robert Gilmore and Luis A. Aguirre (Belo Horizonte), we found a chaotic model from the sunspot numbers. With Jean we got a metastable chaotic model from the records of the Hudson Bay Company about population of Lynxes. I ended this appendix with an application of the techniques developed within the paradigm

of chaos to the heart variability. This was developed with Emeline Fresnel and Emad Yacoub (who were my Ph.D. students).

With Valérie Messager (Rouen), we developed a short autobiography with some psychological insights. We used Otto's astrological chart as guidelines.

With Otto, we revisited the hierarchy of chaos he proposed in the 1983 and the one proposed by Gerold Baier and Michael Klein from Tübingen (who edited a book for Otto's 50th birthday).

I do hope that you will enjoy this book as much as I did myself in editing it and exchanging it with Otto.

Normandie, France
November 2019

Christophe Letellier

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