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Jakub Sawicki

# Delay Controlled Partial Synchronization in Complex Networks

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 Springer

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# Supervisor's Foreword

The focus of this thesis is on delay controlled partial synchronization patterns in complex networks. Synchronization is a ubiquitous phenomenon observed in different disciplines ranging from physics, acoustics, chemistry, biology, neuroscience, and engineering to socioeconomic systems. Synchronization of organ pipes is sometimes wanted and sometimes undesirable. Synchronization of power grids is necessary for a stable and robust operation, and failure may result in cascading power breakdown. Opinion formation in social networks or the stock prices in economic systems provide further examples. Fireflies synchronize their flashing light patterns as part of their mating display. Cardiac pacemaker cells fire synchronously in a regular heartbeat. The synchronization of neurons is believed to play a crucial role in the brain under normal conditions, for instance, in the context of cognition and learning, and under pathological conditions such as Parkinson's disease and epilepsy.

Of particular interest are the interplay of network topology, local nonlinear dynamics, and time-delayed couplings. Time delay is an efficient tool to control synchronization patterns. For example, the finite signal propagation time between coupled neurons significantly influences the dynamics. Besides complete synchronization (in-phase or anti-phase) or cluster and group synchronization, partial synchronization patterns like chimera states, which are characterized by the spatial coexistence of domains with coherent (synchronized) and incoherent (desynchronized) dynamics, are a key issue in current research. Moreover, many recent investigations have focussed upon multilayer systems, where nodes in different layers of the network are coupled in different ways. For instance, different remote layers which are not directly coupled may be synchronized via a relay layer, which itself obeys a different dynamics. This is called relay or remote synchronization.

This thesis aims for a fundamental understanding of these various synchronization phenomena, their interplay with the topology of complex networks, and their control via delayed coupling. It presents theoretical investigations and computer simulations, and is outstanding in its depth and breadth. Simple paradigmatic models are used for the local dynamics of the nodes: the Stuart-Landau model (normal form of a Hopf bifurcation), the Van der Pol oscillator (a generic nonlinear

oscillator), the FitzHugh-Nagumo neuronal model (a paradigmatic model of neuronal excitability), and the time-discrete logistic map (a paradigm of chaos). After a short historical introduction and a concise survey of complex nonlinear systems and networks, the first part deals with single-layer system, and the second part presents multilayer systems.

In Part I of the thesis, at first the simplest configuration consisting of two delay-coupled nonlinear Van der Pol oscillators and its fascinating application to organ pipes, whose synchronization properties depend upon the distance, and hence upon the coupling delay, is considered analytically and numerically. The results offer an explanation of the surprising counter-intuitive nonmonotonic dependence of synchronization upon the distance of the organ pipes which have been found experimentally, but not been understood so far. Then chimera states in a ring network of nonlocally coupled Stuart-Landau oscillators are studied, explaining the chimera mechanism and the transition from classical phase chimeras to coupled amplitude-phase dynamics. Finally, the interplay of time delay and fractal network connectivities in ring networks of Van der Pol or FitzHugh-Nagumo oscillators is investigated, and typical resonance tongues of the chimera patterns are found. Fractal connectivities are motivated by empirical MRI data of the brain, revealing a fractal structure of the brain network.

Part II is concerned with multilayer structures. A neuronal model of the two brain hemispheres based upon empirical structural connectivities of the human brain is studied with the aim of explaining unihemispheric sleep. It has been speculated by many authors that chimera states may be related to unihemispheric sleep observed in migratory birds, aquatic mammals, and to a certain degree even in humans (*first night effect*), but this has never been proved by realistic models. Here for the first time, in computer simulations with empirical brain connectivities and FitzHugh-Nagumo dynamics, it is shown that in certain narrow parameter ranges of the interhemispheric coupling strength, chimera-like states of unihemispheric sleep can be obtained as a partial synchronization pattern. Finally, relay synchronization between remote layers in triplex networks is investigated. In brain research, many examples are known for relay functions of certain areas in the brain, for instance, the hippocampus serves as a relay between the frontal and the visual cortex. For the first time, partial relay synchronization patterns (double chimeras) are found in such a configuration with FitzHugh-Nagumo dynamics, i.e., only the coherent domains of the chimeras in the two remote layers are synchronized, but not the incoherent ones, and the relay layer exhibits completely different dynamics. Various partial synchronization scenarios are found in dependence of the interlayer coupling delay and the parameters of the two layers, and again a resonance tongue structure of the delay control is found. This offers an explanation of partial relay synchronization as observed in real brain experiments of mice. Such scenarios are universal, and they also occur in time-discrete coupled logistic maps. Universal mechanisms for partial relay synchronization can be related to intrinsic pacemakers.

This thesis gives a broad and comprehensive treatment of partial synchronization patterns and their control by time delay in complex networks. It studies and compares various network configurations and dynamic models, and breaks new

grounds by focussing on novel partial synchronization scenarios and pointing out intriguing applications to musical acoustics (organ pipes) and neuroscience (uni-hemispheric sleep, relay effects in brain dynamics).

Berlin, Germany  
September 2019

Prof. Ekehard Schöll

# Abstract

The focus of this thesis are synchronization phenomena in networks and their intrinsic control through time delay. We encounter synchronization everywhere, and it can be a helpful as well as a detrimental mechanism, but definitely, a vital aspect for life on earth. As we are often not aware of it in small systems, it becomes fascinating in larger networks. Nevertheless, in both cases it is based on the same principles, which have been studied in some form for almost four centuries. For the emergence of synchronization, at least two oscillators, characterized by an intrinsic cycle, are necessary, together with a connection between them. We will focus on three generic models, namely the Stuart-Landau, Van der Pol, and FitzHugh-Nagumo system. Starting from a pair of oscillators and proceeding via simple ring networks, this thesis will outline the progression to complex multilayer structures.

Therefore, in Part I, a system of only two coupled oscillators is presented. By numerical and analytical analysis, we show that a seemingly simple system of two organ pipes gives birth to complex bifurcation and synchronization scenarios. Going from a two-oscillator system to a ring of oscillators, states between synchrony and desynchronization are thinkable, where only a part of the system synchronizes. An eminent example of partial synchrony are chimera states. For more than a decade, scientists have tried to solve the puzzle of this spontaneous symmetry breaking emerging on a ring of identical elements. Many previous works have covered various aspects of these states. We provide an analysis of initial conditions, and the understanding of the dependence on these could prove important. The model then is further extended by the addition of time delay, a crucial focus of this thesis. Delay is ubiquitous in the physical world and arises due to necessary propagation or processing time. The arising mathematical difficulties go hand in hand with the role of a powerful control mechanism.

In Part II, we proceed to multilayer systems. We investigate the occurrence of partial synchronization patterns in a neuronal network with empirical structural connectivity. Furthermore, we apply our findings to explain dynamical asymmetry arising from the hemispheric structure in human brain. A particular focus will be on the novel scenario of partial relay synchronization in multiplex networks, a special case of multilayer networks, where the individual layers represent different kinds of



connections. As will be shown, triplex networks allow for synchronization of the coherent domains of chimera states in the outer layers via a remote layer, whereas the incoherent domains remain desynchronized. Finally, we develop an analytical approach and provide an explanation for the so-called double chimera states. Our results have implications for the understanding of partial synchronization in complex networks.

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