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Andreas Herzig · Andrei Popescu (Eds.)

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Editors
Andreas Herzig
CNRS, University of Toulouse
Toulouse, France

Andrei Popescu
Middlesex University London
London, UK

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Preface

These proceedings contain the papers selected for presentation at the 12th International Symposium on Frontiers of Combining Systems (FroCoS 2019). The symposium was held during September 4–6, 2019 in London, UK, at Middlesex University. It was co-located with the 28th International Conference on Automated Reasoning with Analytic Tableaux and Related Methods (TABLEAUX 2019).

FroCoS is the main international event for research on the development of techniques and methods for the combination and integration of formal systems, their modularization and analysis. Previous FroCoS meetings were organized in Munich (Germany, 1996), Amsterdam (The Netherlands, 1998), Nancy (France, 2000), Santa Margherita Ligure (Italy, 2002), Cork (Ireland, 2004, as part of the International Joint Conference on Automated Reasoning, IJCAR), Vienna (Austria, 2005), Seattle (USA, 2006, as part of IJCAR), Liverpool (UK, 2007, co-located with the International Workshop on First-Order Theorem Proving, FTP), Sydney (Australia, 2008, as part of IJCAR), Trento (Italy, 2009), Edinburgh (UK, 2010, as part of IJCAR), Saarbrücken (Germany, 2011), Manchester (UK, 2012, as part of IJCAR), Nancy (France, 2013, co-located with TABLEAUX), Vienna (Austria, 2014, as part of IJCAR), Wrocław (Poland, 2015, co-located with TABLEAUX), Coimbra (Portugal, 2016, as part of IJCAR), Brasilia (Brazil, 2017, co-located with TABLEAUX), and Oxford (UK, 2018, as part of IJCAR). Thus, if we also count the IJCAR editions, in 2019 FroCoS celebrated its 20th edition.

FroCoS 2019 received 30 high-quality paper submissions, which were evaluated by the Program Committee on the basis of their significance, novelty, technical soundness, and appropriateness for the FroCoS audience. Reviewing was single-blind and each paper was subjected to at least three reviews, followed by a discussion within the Program Committee. In the end, 20 papers were selected for presentation at the symposium and publication. We have grouped them in this volume according to the following topic classification: (1) automated theorem proving and model building, (2) combinations of systems, (3) constraint solving, (4) description logics, (5) interactive theorem proving, (6) modal and epistemic logics, and (7) rewriting and unification.

We were delighted to have three outstanding invited speakers. The abstracts of their talks were included in this volume:

- Maria Paola Bonacina: “Conflict-Driven Reasoning in Unions of Theories”
- Stéphane Graham-Lengrand: “Recent and Ongoing Developments of Model-Constructing Satisfiability”
- Uli Sattler: “Modularity and Automated Reasoning in Description Logics”

Uli Sattler’s invited talk was shared with TABLEAUX 2019. Conversely, one of the TABLEAUX invited talks, “Automated Reasoning for the Working Mathematician” by Jeremy Avigad, was shared with FroCoS.

The joint FroCoS/TABLEAUX event had two affiliated workshops:

- The 25th Workshop on Automated Reasoning (ARW 2019), organized by Alexander Bolotov and Florian Kammueller
- Journeys in Computational Logic: Tributes to Roy Dyckhoff, organized by Stéphane Graham-Lengrand, Ekaterina Komendantskaya, and Mehrnoosh Sadrzadeh

It also had two affiliated tutorials:

- Formalising Concurrent Computation: CLF, Celf, and Applications, by Sonia Marin
- How to Build an Automated Theorem Prover – An Introductory Tutorial (invited TABLEAUX tutorial), by Jens Otten

The program committee has offered two awards for outstanding submissions. The Best Paper Award went to "A CDCL-Style Calculus for Solving Non-linear Constraints" by Franz Brauße, Konstantin Korovin, Margarita Korovina and Norbert Müller. The Best Paper by a Junior Researcher Award was shared between "On the Expressive Power of Description Logics with Cardinality" by Filippo De Bortoli as junior co-author and "Verifying Randomised Social Choice" by Manuel Eberl. The awards have been financially supported by Springer.

We would like to thank all the people who contributed to making FroCoS 2019 a success. In particular, we thank the invited speakers for their inspiring talks, the authors for providing their high-quality submissions (all 30 submissions!), revising and presenting their work, the workshop and tutorial organizers for the interesting and engaging events, and all the attendees for contributing to the symposium discussion. We thank the Program Committee members and the external reviewers for their careful, competent reviewing and discussion of the submissions on quite a tight schedule.

We extend our thanks to the local Organizing Committee chaired by Franco Raimondi and to the Middlesex University staff, especially to Nicola Skinner, for offering their enthusiastic support to this event.

We gratefully acknowledge financial support from Amazon, Springer, and Middlesex University. The Association for Symbolic Logic (ASL) has kindly included FroCoS among the events for which students can apply to them for travel funding. Finally, we are grateful to EasyChair for allowing us to use their excellent conference management system.

September 2019

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Abstracts of Invited Talks

Conflict-Driven Reasoning in Unions of Theories

Maria Paola Bonacina

Dipartimento di Informatica, Università degli Studi di Verona
Strada Le Grazie 15, 37134, Verona, Italy
mariapaola.bonacina@univr.it

As the development of automated reasoning has brought to relative maturity multiple reasoning paradigms and tools, a general challenge is that of *interfacing*, *combining*, and *integrating* them, in *reasoning environments* that are more powerful and easier to use. Reasoning in a union $\mathcal{T}$ of theories $\mathcal{T}_1, \ldots, \mathcal{T}_n$ is a context where this challenge arises naturally, and many applications of automated reasoning require to handle a union of at least a few theories. This talk advertises a recent paradigm named CDSAT (*Conflict-Driven SATisfiability*) for *conflict-driven reasoning* in a union of theories [4].

Reasoning in a union of theories can be approached in more than one way. The *equality sharing scheme* by Nelson and Oppen, and its integration in the well-known DPLL($\mathcal{T}$) framework, combine decision procedures for $\mathcal{T}_i$-satisfiability ($1 \leq i \leq n$) into a decision procedure for $\mathcal{T}$-satisfiability. Decision procedures are combined as *black-boxes* that only exchange entailed (disjunctions of) equalities between shared variables. Superposition reasons in a union of theories by taking the union of their axiomatizations: under suitable conditions the termination of superposition is *modular*, so that termination on $\mathcal{T}_i$-satisfiability problems ($1 \leq i \leq n$) implies termination on $\mathcal{T}$-satisfiability problems [1]. *Model-based theory combination* by de Moura and Bjørner is a variant of equality sharing, where the $\mathcal{T}_i$-satisfiability procedures build candidate $\mathcal{T}_i$-models, and propagate equalities true in the current candidate $\mathcal{T}_i$-model rather than entailed. DPLL($\Gamma + \mathcal{T}$) integrates superposition and DPLL($\mathcal{T}$) with model-based theory combination to handle unions mixing axiomatized and built-in theories [5].

DPLL($\mathcal{T}$) and DPLL($\Gamma + \mathcal{T}$) are built around the CDCL (*Conflict-Driven Clause Learning*) procedure for propositional satisfiability (SAT) pioneered by Marques Silva and Sakallah. CDCL builds a candidate partial model of a propositional abstraction of the formula, and applies propositional resolution only to *explain* conflicts between the model and the formula, so that the conflict explanation tells how to update the model and solve the conflict. CDCL inspired several $\mathcal{T}_i$-satisfiability procedures for fragments of arithmetic (e.g, using Fourier-Motzkin resolution only to explain conflicts in linear real arithmetic), and was generalized to first-order logic (without equality) in a theorem-proving method named SGGS (*Semantically-Guided Goal-Sensitive* reasoning) [6]. Methods that perform nontrivial inferences only to explain conflicts are called *conflict-driven*.

In DPLL($\mathcal{T}$) and DPLL($\Gamma + \mathcal{T}$) the conflict-driven reasoning is only propositional as in CDCL: conflict-driven $\mathcal{T}_i$-satisfiability procedures could be integrated only as black-boxes, so that they could not participate in the model construction on a par with CDCL. The MCSAT (*Model-Constructing SATisfiability*) framework by de Moura and Jovanović shows how to integrate CDCL and a conflict-driven $\mathcal{T}_i$-satisfiability procedure, called *theory plugin*, so that *both* propositional and $\mathcal{T}_i$-reasoning are conflict-driven. A key idea is to abandon black-box combination: open the black-box, pull out from the $\mathcal{T}_i$-satisfiability procedure clausal inference rules that can *explain* $\mathcal{T}_i$-conflicts, and enable CDCL and the $\mathcal{T}_i$-plugin to cooperate in model construction.

CDSAT generalizes MCSAT to the multi-theory case, solving the problem of how to combine multiple $\mathcal{T}_i$-satisfiability procedures, some of which are conflict-driven and some of which are black-boxes. The theories are assumed to be equipped with theory inference systems called *theory modules*, with propositional logic viewed as one of the theories in the union. CDSAT provides a framework for the theory modules to cooperate as peers in building a candidate $\mathcal{T}$-model and explaining $\mathcal{T}$-conflicts. Thus, reasoning in a union of theories is achieved by putting together inference systems, rather than procedures or axiomatizations: of course, theory modules are abstractions of decision procedures, and inference rules may correspond to axioms. A black-box $\mathcal{T}_i$-satisfiability procedure is treated as a theory module with only one inference rule that invokes the procedure to check $\mathcal{T}_i$-satisfiability. CDSAT encompasses the previous approaches: it reduces to CDCL if propositional logic is the only theory, to equality sharing if propositional logic is absent and all $\mathcal{T}_i$-satisfiability procedures are black-boxes, to DPLL($\mathcal{T}$) if propositional logic is one of the theories and all other theories have black-box $\mathcal{T}_i$-satisfiability procedures, and to MCSAT if there are propositional logic and another theory with a conflict-driven $\mathcal{T}_i$-satisfiability procedure. Under suitable hypotheses, CDSAT is *sound*, *terminating*, and *complete*.

CDSAT opens several exciting directions for future work, including an integration, or at least an interface, between CDSAT and SGGS, or SGGS enriched with conflict-driven superposition to handle equality. Descriptions of all these approaches appear in recent surveys [2, 3] where the references can be found.

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Recent and Ongoing Developments of Model-Constructing Satisfiability

Stéphane Graham-Lengrand

SRI International

Model-constructing satisfiability is an approach to SMT-solving developed by Jovanović et al., generalising work on satisfiability in non-linear arithmetic [9]. The approach lifts the principles of *Conflict-Driven Clause Learning* (CDCL) from classical propositional reasoning to theory reasoning. It is incarnated by the MCSAT calculus [4, 8] and is implemented in the Yices SMT-solver [5].

Model-constructing satisfiability constitutes a reasoning scheme within the more abstract framework of *conflict-driven satisfiability* (CDSAT) for theory combination [2]. It is more specific than conflict-driven reasoning in that it is tailored to theories that have a standard model, such as arithmetic theories. Using that standard model to evaluate terms and formulae is a central part of model-constructing satisfiability, and allows the reduction of ground satisfiability problems to (series of) interpolation problems, as explained below.

Given some (quantifier-free) *constraints* to satisfy, MCSAT successively guesses assignments of first-order values to first-order variables, with the invariant that none of the constraints evaluates to false, given the assignments made so far. If the invariant can be maintained until all variables are assigned, then the constraints are satisfied by these assignments. But if at any point the invariant cannot be maintained, it means that, for some first-order variable y, every possible choice of value makes one of the constraints evaluate to false. This means that, for a subset $\{C_1,\ldots,C_m\}$ of the constraints with free variables among $x_1,\ldots,x_n,y$, the assignments $\Gamma = \{x_1 \mapsto v_1,\ldots,x_n \mapsto v_n\}$ made so far make the formula $\exists y(C_1 \wedge \cdots \wedge C_m)$ evaluate to false, i.e., $[\![\exists y(C_1 \wedge \cdots \wedge C_m)]\!]_\Gamma = \texttt{false}$. This situation is called a *conflict*, with *conflict constraints* $C_1,\ldots,C_m$.

After hitting a conflict, MCSAT backtracks over some of the guessed assignments and tries new ones. For this, MCSAT requires from the theory solver a symbolic explanation of the conflict, namely a quantifier-free formula I, such that (i) $(\exists y(C_1 \wedge \cdots \wedge C_m)) \Rightarrow I$ is valid in the theory and (ii) $[\![I]\!]_\Gamma = \texttt{false}$. Formula I is the *interpolant of* $(\exists y(C_1 \wedge \cdots \wedge C_m))$ *at* Γ. Any other pick Γ' of assignments such that $[\![I]\!]_{\Gamma'} = \texttt{false}$ will lead to a conflict for the same reason Γ did, so after the backtrack, MCSAT will seek to satisfy the interpolant.

This notion of interpolation relates to *quantifier elimination*, where any formula of the form $(\exists y(C_1 \wedge \cdots \wedge C_m))$ above can be transformed into a quantifier-free formula F such that $(\exists y(C_1 \wedge \cdots \wedge C_m)) \Leftrightarrow F$ is valid in the theory. Property (i) of interpolation is weaker than such an equivalence, and property (ii) makes the production of the interpolant "model-driven", i.e., driven by assignments Γ.

MCSAT applies for instance to linear and non-linear real arithmetic, where the interpolants are respectively produced using Fourier-Motzkin resolution and Cylindrical Algebraic Decomposition (CAD). These key mechanisms of quantifier elimination are used in MCSAT on demand, in response to a particular conflict.

MCSAT is also being applied to the theory of bit-vectors. The difficulty there is the diversity of bit-vectors operations that may occur in conflict constraints. While bit-blasting provides a default interpolation mechanism, the interpolants are not very good for the efficiency of MCSAT, being closer to the bit level than to the word level. Current research and implementation efforts in Yices aim at better interpolants, when the conflict constraints lie within some suitable fragments of the bit-vector theory, for instance in linear arithmetic modulo [7].

Following work on the application of SMT-solving to intuitionistic propositional reasoning [3, 6], ongoing research also applies the MCSAT approach there, using the worlds of a Kripke model in the assignments of values to variables.

Finally, the connection with quantifier elimination suggests the generalisation of MCSAT to quantified problem. We are currently developing this generalisation, in connection with previous work on quantified satisfaction [1].

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Modularity and Automated Reasoning in Description Logics

Uli Sattler

School of Computer Science, University of Manchester, UK
uli.sattler@manchester.ac.uk

Description Logics [2] are decidable fragments of first order logics closely related to modal logics and the guarded fragment. Through their use as logical underpinning of the Semantic Web Ontology language OWL [5], they are now widely used in a range of areas. As a further consequence, DL reasoners have to deal with logical theories—called ontologies—of increasing size and complexity, and domain experts using DLs ask for increasingly sophisticated tool support. One of the many areas that have been considered in this aspect is *modularity* [4, 8], a concept that has successfully been used to tame complexity and enable separation of concerns in other areas, in particular Software Engineering.

Firstly, we consider the task of extracting, from one ontology, a small/suitable fragment that captures a given topic, usually described in terms of its signature. The question of suitability versus size here is interesting, and has given rise to different notions of modules and their properties and algorithms for their extraction [1, 4, 6, 10–12, 15, 16]. Secondly, it would be extremely useful if we could "modularise" a large ontology into suitable coherent fragments (OWL has an "imports" construct that supports some kind of modular working with/storage of an ontology) [7, 9, 13]. Thirdly, if we have such a nice, modular ontology, the question arises of how a group of domain experts can work independently on these without undesired side effects. Fourth and finally, reasoning over ontologies is often a highly complex task, and a natural question arising is whether/which form of modularity can be used and how to optimise reasoning [3, 14, 18, 19].

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