

# Lecture Notes in Electrical Engineering

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Paolo Di Barba · Sławomir Wiak

# MEMS: Field Models and Optimal Design

 Springer

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