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Kurt E. Oughstun

Electromagnetic and Optical Pulse Propagation

Volume 1: Spectral Representations in
Temporally Dispersive Media

Second Edition



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*The cautious guest
who comes to the table
speaks sparingly.
Listens with ears
learns with eyes.
Such is the seeker of
knowledge.*

Hávamál

The Sayings of the Vikings

Translated by Björn Jónasson

*This volume is dedicated to
Professor John B. Bulman
Professor George C. Sherman
Professor Emil Wolf
Professor Kenneth I. Golden
Professor Gagan Mirchandani
Dr. Walter J. Fader
Dr. Edward A. Sziklas
Dr. Richard Albanese
My teachers, mentors, and valued colleagues.*

Preface to the Second Revised Edition

This revised second edition of the first volume of this two-volume text on time- and frequency-domain electromagnetics in dispersive attenuative media contains most of the same material presented in the first edition with minor editing and correction where necessary, as well as expanded development of several deserving topics. These include sections on the aberration of light and the relativistic Doppler effect, the conjugate electromagnetic fields and invariants, the near field of an extended time-harmonic linear dipole source, the molecular theory of reflection and refraction, the Ewald–Oseen extinction theorem, the Fresnel reflection and transmission coefficients for complex lossy media, the properties of metamaterials, and the Sherman expansion of source-free wave fields. In addition, the majority of the material presented in Chap. 9 of Volume 2 has been moved to Volume 1 as it is more appropriate for it to be included with the topics presented in this volume.

As now rewritten, Chaps. 1–5 provide a detailed development of the fundamental properties of and interrelationship between microscopic and macroscopic electromagnetic field theories. There is sufficient material here for a one-semester senior or first year graduate-level course in electromagnetic theory, including a variety of challenging problems at the end of each chapter. The remaining Chaps. 6–9, taken together with selected topics from Volume 2, would then form the basis for an advanced graduate-level course in electromagnetic wave theory.

My research on this topic began in the early 1970s when I was a graduate student at The Institute of Optics of The University of Rochester with George C. Sherman as my advisor and fellow, albeit more senior graduate students, Anthony J. Devaney and Jakob J. Stamnes as mentors. Constructive criticisms regarding several topics appearing in the first edition of this volume from Richard Albanese, Natalie Cartwright, and Christopher Palombini are gratefully acknowledged. In addition, I am indebted to the physics and engineering students who have successfully taken my undergraduate and graduate electromagnetic theory courses at the University of Vermont for pointing out some of the all too many typographical errors appearing in

the first edition of this text. In spite of this, I blissfully remain a “two-finger” typist. Dyslexia doesn’t help.

Burlington, VT, USA
January 2017

Kurt E. Oughstun

Preface to the First Edition

This two-volume text presents a systematic theoretical treatment of the radiation and propagation of pulsed electromagnetic and optical fields through temporally dispersive attenuative media. Although such fields are often referred to as transient, they may be short-lived only in the sense of an observation made at some fixed point in space. Because of their unique properties when the pulse spectrum is ultrawideband with respect to the material dispersion, specific features of the propagated pulse are found to persist in time long after the main body of the pulse has become exponentially small. Therein lies both their interest and usefulness.

The subject matter divides naturally into two volumes. Volume 1 presents a detailed development of the fundamental theory of pulsed electromagnetic radiation and wave propagation in causal linear media that are homogeneous and isotropic but which otherwise have rather general dispersive, absorptive properties. The analysis is specialized in Volume 2 to the propagation of electromagnetic and optical pulses in homogeneous, isotropic, locally linear media whose temporal frequency dispersion is described by a specific causal model. Dielectric, conducting, and semi-conducting material models are considered. These two volumes present sufficient material to cover a two-semester graduate sequence in electromagnetic and optical wave theory in physics and electrical engineering as well as in applied mathematics. Prerequisite material includes undergraduate courses in electromagnetic field theory and complex variable theory. Challenging problems are given throughout the text.

The development presented in Volume 1 provides a rigorous description of the fundamental time-domain electromagnetics and optics in linear temporally dispersive, absorptive media. The analysis begins with a detailed review of the classical Maxwell–Lorentz theory and the invariance of the field equations in the special theory of relativity. The macroscopic theory is then obtained from the microscopic theory through a well-defined spatial-averaging procedure. A general description of macroscopic electromagnetics and the role that causality plays in the constitutive (or material) relations are then developed. The description of electromagnetic energy flow in causally dispersive media is then developed based on the conservation laws for the electromagnetic field. The angular spectrum of plane wave representation of the pulsed radiation field in homogeneous, isotropic,

locally linear, temporally dispersive media is derived and expressed in the form of the classical integral representations of Weyl, Sommerfeld, and Ott. The theory is then applied to the general description of pulsed electromagnetic and optical beam fields in dispersive materials, showing how the effects of temporal dispersion and spatial diffraction are coupled.

The theory presented in Volume 2 provides the necessary mathematical and physical basis to describe and explain the detailed dynamical evolution of a pulse as it travels through a linear temporally dispersive, absorptive medium. This is the subject of a classic theory with origins in the seminal research by Arnold Sommerfeld and Leon Brillouin in the early 1900s for a Lorentz model dielectric and described in modern textbooks on advanced electrodynamics. This classic theory has been carefully reexamined and extended by George Sherman and myself over the time period from 1974 to 1997 using modern asymptotic methods of approximation. We have developed a physical model that provides a straightforward quantitative algorithm that not only describes the entire dynamical field evolution in the mature dispersion regime but also explains each feature in the propagated field in simple physical terms. This model reduces to the group velocity description in the weak dispersion limit as the material loss approaches zero. The uniform asymptotic description was then completed by Natalie Cartwright and myself in 2007. Finally, the persistent controversy regarding the question of superluminal pulse propagation in dispersive media is examined in light of the recent results establishing the domain of applicability of the group velocity approximation.

My research in this area began in the early 1970s when I was a graduate student at The Institute of Optics of The University of Rochester in Rochester, New York. I am grateful for the financial support during that critical period by The Institute of Optics, the Corning Glass Works Foundation, the National Science Foundation, and the Center for Naval Research. This research continued while I was at the United Technologies Research Center, the University of Wisconsin at Madison, and, finally, the University of Vermont with an extended sabbatical at the Universitetet i Bergen in Norway that was generously supported by the Norwegian Research Council for Science and the Humanities. The critical, long-term support of this research by both Dr. Arje Nachmann of the United States Air Force Office of Scientific Research and Dr. Richard Albanese of the Air Force Research Laboratory is gratefully acknowledged.

A diverse variety of textbooks has contributed to my understanding of both electromagnetic and optical field and wave theory. Chief among these are J. D. Jackson's *Classical Electrodynamics*, M. Born and E. Wolf's *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, and J. M. Stone's *Radiation and Optics: An Introduction to the Classical Theory*. Each has had a fundamental influence on my own research.

Burlington, VT, USA
January 2010

Kurt E. Oughstun

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