Part V
Advanced Applications in Physics and Chemistry

Finally, we combine both linear algebra and group theory in practical applications in quantum chemistry and general relativity. First, we use group theory to introduce the permutation group and study the determinant of a square (complex) matrix. Once the determinant is used in our quantum-mechanical model, we can write the expected energy and obtain the Hartree–Fock system: a pseudo-eigenvalue problem. Thanks to linear algebra, the (generalized) eigenvectors have a desirable property: orthogonality. This is indeed how group theory and linear algebra combine to form a complete theory with practical applications.

We conclude with an interesting application in general relativity: Einstein equations. To introduce them, we must use new features in linear algebra: tensors and algebraic operations between them. For this purpose, we must introduce a new principle: Einstein's summation convention. It improves on the standard sums used in linear algebra. In fact, it tells us how to raise and lower indices and come up with a coherent summation strategy. Thanks to it, the nonlinear system of equations gets particularly easy to introduce.