

Computational Modeling of Tensegrity Structures

Buntara Sthenly Gan

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*To
Rie, Kenta, Sūchan, and Shichan
Thank you for the love and encouragement
over the years*

Preface

The concept of tensegrity as a structure has been around since the middle of the twentieth century and is currently seeing an enormous growth of interest. From the early attempts into a new form of sculpture, it is now incorporated into the art and architectural form and deployable structures in space and is attracting the attention of biologists to reveal the mechanism of a cell, the smallest part of the living creature. I myself am not a biologist or biochemist working in this exciting field but am interested in the mechanics of biomolecules and biostructures.

I have always been impressed by Prof. Lee Jaehong of Sejong University, who has published plenty of papers in high-rank journals on the tensegrity topics. I feel envy to what he has achieved so far. Before writing this book, I was having a free talk with him during my sabbatical leave at Sejong University in the summer of 2016. He said that he has been doing researches on the tensegrity topics for so long, but there is not a good book on the tensegrity for educational purpose. I took the challenges inspired by him by attempting to write this book.

This book is intended for students in engineering and biological science who are beginning a series of courses in structures and mechanics, for practitioners in designer and consultant companies who are starting to learn or find out how to analyze the tensegrity structures, and for researchers who are going to pursue or extend their research in every aspect of tensegrity structures.

This book is written to be as a textbook or workbook with examples on how to solve the problems by understanding the basics of linear algebra, geometry, and matrix operations that are necessary for the physical mechanisms of tensegrity. All the formulations of concepts are accompanied by MATLAB program lists which are intended for easy understanding and learning how the formulas work. Just like in the computer science, we know the modern editor software now has been adopting the “WYSIWYG” system in which the displayed text can be edited in a form exactly resembling its appearance when printed on a paper.

As for the organization of the book, in Chap. 1, the definition, mathematical modeling concept, and classification of tensegrities are presented. Inside it, step-by-step instructions which have been developed by the author and used to educate students on how to create a tensegrity in a workshop and class are given and freely

can be used for educational purposes. In Chap. 2, the selected linear algebra which is the basic requirement to understand the computational tensegrity is presented with examples, including the program listings. Chapter 3 introduces the singular value decomposition (SVD) for computations of structures. Force density method which is widely used in the analysis of tensegrity is discussed. Through examples and program lists, the classification of the structural system is shown. The form-finding of tensegrity processes are discussed in depth including applications in Chap. 4. Chapter 5 demonstrates how to design a tensegrity in the real world, starting from form-finding, structural designs, and computational methods which are given in detail. In Chaps. 6, 7, and 8, the applications of form-finding and analyzing the tensegrity in art, architecture, mechanics, and biology are shown including their MATLAB codes.

The writing of this book was an enjoyable experience for myself. I received benefits from my professional works, encouragement, and supports from many colleagues as well as my students who have taught me how to explain to them difficult concepts in modest expressions. While it is not possible to name all of them, without their help and supports, I could not have been possible to finish the writing of this book.

My sincere thanks are due to my respected former Emeritus Professor Fumio Nishino in the University of Tokyo for his philosophies in teaching and research which have been useful in my professional life. Great appreciations are due to Professor Mitsuharu Kurata in Nihon University for constantly motivating me to find the “truth” in science. I wish to thank Dr. Nguyen Dinh Kien, my research colleague in VAST, for his fruitful ideas, encouragements, and supports in my research works. I am indebted to Prof. Han Ay Lie in Universitas Diponegoro for her continuous encouragements and inspiring ideas in doing my research works. I am also grateful to Prof. Lee Jaehong, Prof. Lee Seunghye, and Prof. Kim Nam Il at Sejong University for their hospitality, collaborations, and many stimulating discussions concerning several topics during my sabbatical leave. I want to thank my former master students in line of this research works, namely, Mr. Manabu Yamamoto and Mr. Shota Kiryu, who have been working hard in helping me to modify and improve the program and work out some examples. I appreciate much the help and patience from Senior Publishing Editor Mr. Michael Luby, Project Coordinator Ms. Kiruthika Kumar, Production Editor Mr. Menas Donald Kiran and Project Manager Ms. Abitha Pradeep Coumar during the preparation and publication of this book.

The readers are welcomed to use the MATLAB program and function lists freely. However, the author does not guarantee that the program and function lists are without any error. The author is also not responsible for any damage or loss by using the

program or function lists. The reader is fully responsible for verifying before using the program or function lists.

Most books are not free from errors, especially those with many mathematical equations and numbers. I wish to thank in advance those readers who are willing to draw attention to any typos and errors using the following e-mail address:

[buntara@arch.ce.nihon-u.ac.jp].

Koriyama, Japan
February 2019

Buntara Sthenly Gan

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