

# Encountering Algebra

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Editors

# Encountering Algebra

A Comparative Study of Classrooms in  
Finland, Norway, Sweden, and the USA

 Springer

*Editors*

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# Preface

This volume builds on a collaborative research project about algebra learning in four countries, Finland, Norway, Sweden, and the USA (California). The idea of the project was to document and analyze the first lessons when students are introduced to algebra. In concrete terms, each country team contacted teachers and asked them for permission to video record the lessons when they introduced the curricular unit of algebra. The choice of the topic of algebra was motivated not only by our joint interest in teaching and learning in this domain, but also by the observations reported in much of the literature that algebra is a hurdle for many students. Concepts such as variable, unknowns, and equivalence and the solving of equations represent activities that many find challenging, and the aim of the project has been to see how instruction is organized, and how students approach, struggle with, and appropriate basic algebraic symbols and modes of thinking. This implies that the spirit of the project has been to explore the perspectives and rationalities of the participants, students, and teachers, as they engage in algebra in regular instructional settings.

As the reader of this volume, and other reports that have been produced in the project, will see, there are both similarities in and variations between the ways in which algebra is introduced in the classrooms we have documented. We are well aware that our empirical materials do now allow us to make strong generalizations in a statistical sense about how algebra is introduced in different countries. The specific background of this volume is that during the extensive discussions over several years in the distributed project group about our data and analyses, we decided to try to pick out elements of introducing algebra that we found characteristic of our respective materials, i.e., elements that stood out as characteristic when compared to what we saw in the materials from the other countries. Thus, each team was encouraged to select one topic or feature of the teaching and learning documented that they perceived as typical of their own educational traditions and of how algebra is introduced in the lessons documented. The empirical chapters (4-7) present these case studies from the respective countries. In the case of Sweden, the focus is on how children participate in teaching and learning when algebra is introduced. In the case of Norway, the focus is on the nature of tasks and

examples that teachers use and produce to help clarify basic algebraic concepts and modes of reasoning. In the chapter from Finland, the ways in which students approach equation solving are explored, and in the US chapter consistencies between teachers' conceptions of what it means to learn mathematics/algebra, on the one hand, and the instruction they engage in, on the other hand, are analyzed. Chapter 8, the final empirical section, reports a comparative analysis of how students in the four different countries solve a patterning task given after the first four lessons of algebra. A patterning task, thus, potentially involves modeling and using algebraic reasoning in order to make mathematical sense of a situation where students have to oscillate between, on the one hand, concrete observations of a pattern that they can see in front of them and, on the other hand, attempts to represent this pattern in mathematical terms. The task itself is taken from an international comparative study of mathematics achievement, and it proved to be an interesting test bed for studying learning trajectories.

The project itself has been an exciting and truly international collaboration between the four teams. Representatives of teams have met physically on a few occasions, but most of the collaboration has taken place through videoconferencing where we have taken decisions on how to proceed with the research at various stages and where we have discussed our data, analyses, and findings. This work has been conducted in real time, which has meant that the members of the team in California had to be ready for academic exercises very early in the morning, while the Finnish team members had to stay in their offices after regular working hours. The fact that it is possible to conduct research seminars under these conditions, including activities such as projecting data to be discussed on a shared screen and scrutinizing analyses suggested, has been both rewarding and inspirational. And we are very satisfied that this idea, built into the design of the project, turned out to work very well technically as well as academically.

The project was originally initiated as a Nordic research project by Roger Säljö, Sweden, Maria Luiza Cestari, Norway, and Ole Björkqvist, Finland, in collaboration with Jim Stiegler, USA. The practical work of collecting empirical data, transcribing, coding, and analyzing was conducted in each country by a team of researchers. In addition to all those participating as authors in this book, we would like to acknowledge the contributions of Rimma Nyman, Anna Lundberg, and Elisabeth Rystedt, three PhD students who joined the project at different times, providing input and new perspectives. Our special thanks go to all the teachers who opened their classrooms for us, allowing us to document their practices on video, and who also willingly took part in individual and focus group interviews.

The project has been funded by the Joint Committee for Nordic Research Councils for the Humanities and Social Sciences (grant 2135-08-210321), and we are very grateful for this possibility to do comparative research in algebra learning, while at the same time testing the possibilities for academic work across time and space. The research teams would also like to express their thanks to the administrative staff at the four universities for their support. The core person in the complicated sub-project of handling resources, overheads, currency fluctuations, and other

facts of academic life has been Mrs. Doris Gustafson at the University of Gothenburg. Doris Gustafson has an incredible experience in the administrative sides of research collaborations, national as well as international, and without her skilled support and problem-solving capacities we would not even have been able to put together a coherent application.

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